On the mobility of a robot-trajectory process

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Abstract. Discussing the dynamic capabilities of a revolute joint arm robot performing some given trajectory requires a particular consideration of all constraints of robot and process as prescribed by the trajectory. It is possible to transform the equations of motion to a quadratic form allowing some statements referring to the mobility of the robot-trajectory-process.

Introduction. For establishing the quadratic form of the equations of motion three steps are needed: First, apply a multibody formulation of the Lagrange I form, second, eliminate the constraint forces by projecting these relations from Cartesian world coordinates $\boldsymbol{z}$ to robot minimal coordinates $\boldsymbol{q}$ and third, project in addition the minimal coordinate dynamics onto the trajectory curvilinear coordinate s. This requires the following equations:

$$\boldsymbol{M}\ddot{\boldsymbol{z}} + \boldsymbol{f}^g - \boldsymbol{f}^e - \boldsymbol{f}^c = 0, \qquad \ddot{\boldsymbol{\Phi}} = \boldsymbol{W}^T\ddot{\boldsymbol{z}} + [(\frac{d\boldsymbol{W}^T}{dt})\dot{\boldsymbol{z}} + (\frac{d\bar{\boldsymbol{w}}}{dt})] = \boldsymbol{W}^T\ddot{\boldsymbol{z}} + \hat{\boldsymbol{w}}, \qquad \boldsymbol{z} \in \mathbb{R}^{n_z},$$

$$\dot{\boldsymbol{\Phi}} = \boldsymbol{W}^T\dot{\boldsymbol{z}} + \bar{\boldsymbol{w}}, \quad (\frac{\partial\dot{\boldsymbol{\Phi}}}{\partial\dot{\boldsymbol{q}}}) = \boldsymbol{W}^T(\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}}) = (\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})^T\boldsymbol{W} = \boldsymbol{0}, \qquad \boldsymbol{f}^c = -\boldsymbol{W}(\boldsymbol{z},t)\boldsymbol{\lambda},$$

$\boldsymbol{\Phi}$ are the constraints and $\boldsymbol{f}^c$ the constraint forces, the rest self-explaining. Multiplying the first equation with $(\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})^T$ thus eliminating the constraint force we come to the minimal form

$$\boldsymbol{M}_q\ddot{\boldsymbol{q}} + (\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})^T\boldsymbol{M}\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}} + (\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})^T(\boldsymbol{f}^g - \boldsymbol{f}^e - \boldsymbol{f}^a - \boldsymbol{f}^p) = \boldsymbol{0}, \qquad \boldsymbol{M}_q = [(\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})^T\boldsymbol{M}(\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})].$$

These equations are projected onto the path coordinate s by $\frac{dq}{ds} = \boldsymbol{q}'$, $\ddot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{q}'(\dot{s}^2)' + \boldsymbol{q}''(\dot{s}^2)$ and $\ddot{s} = \frac{d\dot{s}}{dt} = (\frac{d\dot{s}}{ds})\dot{s} = \frac{1}{2}(\dot{s}^2)'$ coming out with the quadratic form

$$\boldsymbol{A}(s)(\dot{s}^2)' + \boldsymbol{B}(s)(\dot{s}^2) + \boldsymbol{C}(s) = \boldsymbol{T}(s), \qquad (\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{T}) \in \mathbb{R}^{n_q},$$

$$\boldsymbol{A}(s) = \frac{1}{2}\boldsymbol{M}_q\boldsymbol{q}', \quad \boldsymbol{B}(s) = \boldsymbol{M}_q\boldsymbol{q}'' + \boldsymbol{H}_q(\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})^T\boldsymbol{M}[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})\boldsymbol{q}']\boldsymbol{q}' + \boldsymbol{W}_q(\boldsymbol{W}_q^T\boldsymbol{M}_q^{-1}\boldsymbol{W}_q)^{-1}[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial \boldsymbol{z}_0}{\partial \boldsymbol{q}})\boldsymbol{q}']\boldsymbol{q}',$$

$$\boldsymbol{C}(s) = \boldsymbol{H}_q\boldsymbol{f}_q^e, \quad \boldsymbol{T}(s) = \boldsymbol{H}_q\boldsymbol{f}_q^a, \quad \boldsymbol{f}_q = (\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}})^T(\boldsymbol{f}^g - \boldsymbol{f}^e - \boldsymbol{f}^a - \boldsymbol{f}^p)_q, \qquad \boldsymbol{H}_q = [\boldsymbol{E}_n - \boldsymbol{W}_q(\boldsymbol{W}_q^T\boldsymbol{M}_q^{-1}\boldsymbol{W}_q)^{-1}\boldsymbol{W}_q^T\boldsymbol{M}_q^{-1}],$$

The relations above define for each set of the driving forces $(\boldsymbol{T}_{max}, \ \boldsymbol{T}_{min})$ a set of two straight lines in the plane $[(\dot{s}^2)' \ \ (\dot{s}^2)$ for s=constant] for each robot degree of freedom. Considering the intersections of these straight lines results in an area as envelope of all these lines, for each trajectory point s=constant. Putting these areas together for all points s results in a space limited by ruled surfaces. Only within this space or on the surface motion can take place. It represents an excellent tool for design considerations. The Figure depicts the

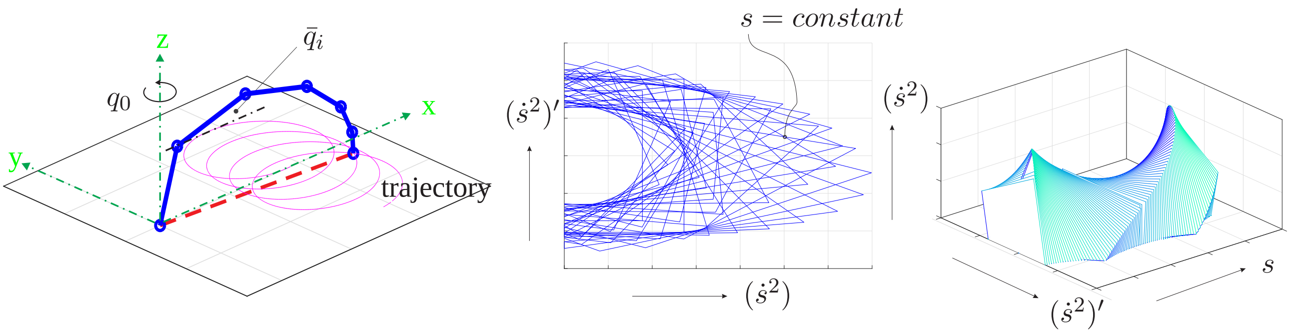

Figure 1: Robot/trajectory combination and some Results

principal situation. The robot follows a trajectory, and the results are evaluated applying the above relations. The numerical results shown are for a robot with two arms following a horizontal circle. The middle pictures are the allowed motion areas arranged along the path for various s. The right picture illustrates the motion space formed by ruled surfaces by ordering the areas of the middle picture along s. Applications will be discussed.

References

[1] F. Pfeiffer, *Optimal Trajectory Planning for Manipulators*, Systems and Control Encyclopedia, Pergamon Press, Oxford, New York, 1990