On the computational efficiency of pseudo-arclength path-following schemes for multiple DOFs dynamical systems

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Abstract. We discuss the computational efficiency of an enhanced version of a pseudo-arclength path-following scheme to construct frequency response curves of multi-degree-of-freedom nonlinear dynamical systems. The path-following approach is based on the Poincaré map, that allows to tackle nonautonomous systems with discontinuous vector fields. The key point of the present implementation concerns the increased convergence speed when using the Jacobian matrix of the problem that serves as iteration matrix within the modified Newton-Raphson scheme.

Introduction

Pseudo-arclength continuation is a well-known, powerful approach to path-follow numerically the curves of the periodic solutions of nonlinear dynamical systems tackled by searching for the fixed points of a suitable Poincaré map. Several numerical tools are available to obtain continuation curves, often presented as open-source softwares or toolboxes compatible with MATLAB such as AUTO, MATCONT, COCO, to cite but only a few. However, each of them is rarely employed to study large-scale systems, due to the need of numerous time integrations that drastically slow down the algorithm speed [1]. A recent review paper argued how the computational costs of the method have limited its application to real-world problems [2].

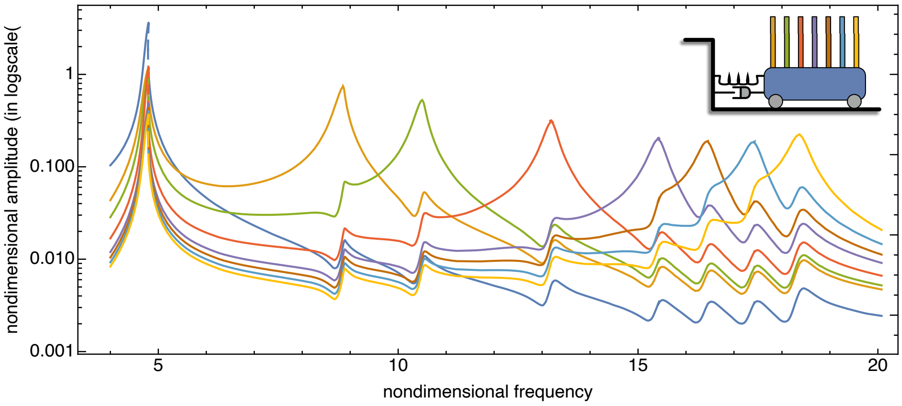

Figure 1: Frequency response curve of a system composed by a shuttle mass and 7 parallel nonlinear beams, characterized by an increasing stiffness ratio, from left to right.

Scope and Results

The present work discusses the implementation of an *ad hoc*, in-house developed, `c++` code able to improve the whole efficiency of the pseudo-arclength path-following scheme, achieved both in the reduction of the computational burden as well as in the numerical accuracy.

The proposed formulation tackles, in particular, the numerical evaluation of the Jacobian matrix associated with the Poincaré map [3], as this is definitely the most expensive computational task (cost being proportional to DOFs×DOFs time-integrations over one period). Moreover, this is the key computational step to improve both accuracy and convergence. Furthermore, we propose a Krylov-Subspace accelerated algorithm [4] that, combined with a modified Newton-Raphson scheme using the factorized version of the Jacobian matrix, largely speeds up the iterative evaluations of the solution points.

Some benchmark tests with a significantly high number of DOFs are presented, so as to emphasize the performance of the developed code (see, for instance, Figure 1).

References

[1] Detroux T., Renson L., Kerschen G. (2014) The Harmonic Balance Method for Advanced Analysis and Design of Nonlinear Mechanical Systems. In: Kerschen G. (eds) Nonlinear Dynamics, **2**:19–34. Springer, Cham.

[2] Renson L., Kerschen G., Cochelin B. (2016) Numerical computation of nonlinear normal modes in mechanical engineering. *Journal of Sound and Vibration* **364**:177–206.

[3] Lacarbonara W. (2013) Nonlinear Structural Mechanics Theory, Dynamical Phenomena, and Modeling. Springer, New York.

[4] Scott M. H., Fenves G. L. (2010) Krylov Subspace Accelerated Newton Algorithm: Application to Dynamic Progressive Collapse Simulation of Frames. *Journal of Structural Engineering – ASCE* **136**:473–480.