Modeling the fear induced spatiotemporal dynamics of three-species agroecosystems

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Abstract. In the present work, we proposed a tritrophic food chain model of wolf spiders, insect pests, plant community interactions incorporating the cost of fear in the predation rate of insect pests. We studied the boundedness, local, and global stability of the proposed system. Spatiotemporal extension of the proposed system is also discussed. Turing instability conditions are obtained using the linear stability analysis. Various numerical simulations have been performed to validate the analytical findings and to better explore the system dynamics. It is observed that the level of fear has a stabilizing impact on system dynamics. Interesting Turing patterns have been obtained and it is found that the level of fear and diffusion coefficients have a significant effect on the spatial patterns.

Introduction

Herbivory can be very overpricing as it significantly removes the valuable photosynthesis material of plants required for the growth and reproduction of plants. Every year millions of dollar losses occur in the agricultural industry due to herbivory. Usually, pesticides are used to reduce herbivory. These synthetic chemicals have an adverse effect on the environment. For environment-friendly concern, utilizing natural methods for controlling pest density is the need of the day. In the agricultural ecosystem wolf spider of family Lycosidae are often the dominant predatory arthropods and a good candidate for biological control measures. The presence of spiders in agroecosystem impact insect pests not only through direct predation but also through the fear effects. Fear effects in predator-prey interaction models are first incorporated by Wang et al. [1]. After that, various studies [2, 3] have been carried out. In the present work, we proposed a three-species food chain model for wolf spiders, insect pests and plant community in an agroecosystem. In which, we have introduced the cost of fear due to wolf spiders in the foraging activities of insect pests. Our objective is to explore the cascading effect of fear of top predator on the lower trophic level. The mathematical model is given by the following system:

$$\begin{aligned}
\frac{\partial P}{\partial t} &= r_1 P\left(1-\frac{P}{K_1}\right) - \frac{\omega P I}{(1+kS)(P+D)} + d_1\Delta P, \quad t>0,\ (x,y,z)\in\Omega,\\
\frac{\partial I}{\partial t} &= -(\delta+m)I + \frac{\omega_1 P I}{(1+kS)(P+D)} - \frac{\omega_2 I^2 S}{(I^2+D_1^2)} + d_2\Delta I, \quad t>0,\ (x,y,z)\in\Omega,\\
\frac{\partial S}{\partial t} &= r_2 S\left(1-\frac{S}{K_2}\right) + \frac{\omega_3 I^2 S}{(I^2+D_1^2)} + d_3\Delta S, \quad t>0,\ (x,y,z)\in\Omega,
\end{aligned} \tag{1}$$

with the initial conditions $P(0,x,y,z)>0,\quad I(0,x,y,z)>0,\quad S(0,x,y,z)>0,\ (x,y,z)\in\Omega$ and the zero-flux boundary conditions

$$\frac{\partial P}{\partial \nu} = \frac{\partial I}{\partial \nu} = \frac{\partial S}{\partial \nu} = 0, \qquad t>0,\ \ (x,y,z)\in\partial\Omega. \tag{2}$$

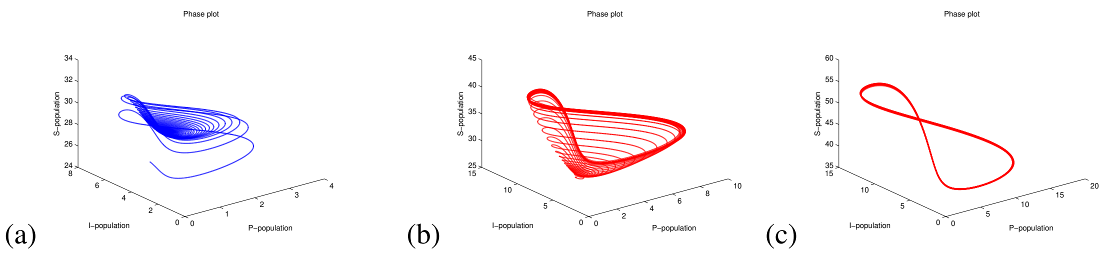

Figure 1: Phase portraits of system with parameters $r_1=1.2, K_1=100, \omega=5.89, D=10, k=0.02, \delta=0.05, \omega_1=2.58, \omega_2=0.25, D_1=20, r_2=0.05, K_2=20, \omega_3=0.8$. (a) $m=0.001$, stable (b) $m=0.05$, stable limit cycle (c) $m=0.1$, limit cycle.

Discussion and Conclusion

One and two-parameter bifurcation diagrams have been plotted and the existence of a stable limit cycle, subcritical and supercritical Hopf-bifurcations have been investigated. It is observed that the level of fear k has a stabilizing impact on the system dynamics. It is also observed that insect pests become less sensitive towards perceived predation risk for the high value of conversion rate. Various interesting Turing patterns have been found and it is observed that the level of fear and diffusion coefficients have a significant impact on these spatial patterns.

References

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