Continuous Leaderless Synchronization Control of Multiple Spacecraft on $SO(3)$

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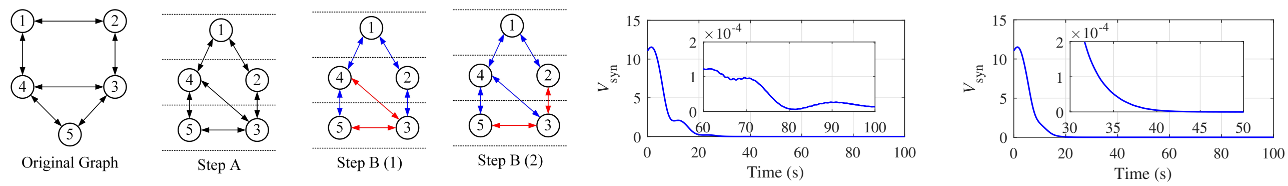
Abstract. This paper presents a solution to the leaderless consensus of multiple spacecraft on $SO(3)$ under a connected undirected graph. An algorithm is proposed to generate an undirected tree graph from the communication topology of concern. A distributed observer is designed to estimate the desired attitude and angular velocity for each spacecraft under the generated tree graph. Under a general connected undirected graph, an adaptive controller is developed to finish the synchronization mission. Both the cases with zero and non-zero final angular velocities are considered. Theoretical proof and numerical simulations are given to show the effectiveness of the proposed controllers.

Introduction

Attitude coordination is one key to some spacecraft formation flying missions. However, since the configuration space of the spacecraft attitude is the special Lie group $SO(3)$, the strongly nonlinear attitude dynamics poses great challenges for the controller design. To avoid the singularities of Euler angles and Modified Rodriguez Parameters (MRPs) and the ambiguity associated with quaternions, some control algorithms have been developed directly on $SO(3)$. However, no continuous time-invariant control laws can globally stabilize the attitude maneuvering and almost global asymptotic stability is the best result because $SO(3)$ is not diffeomorphic to any Euclidean space. Considering that the commonly used actuators, such as reaction wheels and control moment gyroscopes, can only provide continuous torques, this paper will try to present a continuous controller. In the distributed tracking of a leader, several observer-based methods have been proposed. However, the leader-follower architecture relies heavily on the leader. The error or loss of a leader will cause the mission's failure. The leaderless consensus strategy can avoid this problem. Hence, it is necessary to consider the leaderless consensus of multiple spacecraft. However, all existing studies on leaderless attitude consensus are based on specified communication topologies or with local stability [1]. The available methods [2, 3] with a distributed observer to estimate the leader's information do not work in the case of the leaderless consensus under a general undirected graph. Hence, the leaderless attitude synchronization on $SO(3)$ under a general connected undirected graph is still unsolved. This work will provide a solution to such an open problem.



(a) Generating a tree graph using Algorithm 1 (b) Synchronization error (Case I) (c) Synchronization error (Case II)

Figure 1: An example of generating a tree graph using Algorithm 1 and synchronization errors in two simulation cases

Results and Discussion

As shown in Fig. 1(a), to facilitate the design of a distributed controller, an algorithm is proposed to generate an undirected tree graph based on the local information exchange only. To provide the reference signal for each spacecraft, distributed observers are designed under the resulting undirected tree graph. Two continuous control algorithms are presented to achieve zero or non-zero final angular velocity, respectively. The stability conditions are given based on the rigorous theoretical analyses. Figs. 1(b) and 1(c) give the simulation result in cases with non-zero and zero final angular velocity, respectively. From Figs. 1(b) and 1(c), it can be concluded that the proposed controllers can finish the leaderless synchronization missions efficiently in both cases. Since these two continuous controllers are developed directly on $SO(3)$, the singularities and ambiguities associated with other attitude representations can be avoided and the controllers' continuity facilitates its implementation in practice.

References

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