From Chaotic Dynamics to Periodic Dynamics: Noise Assisted Response Steering

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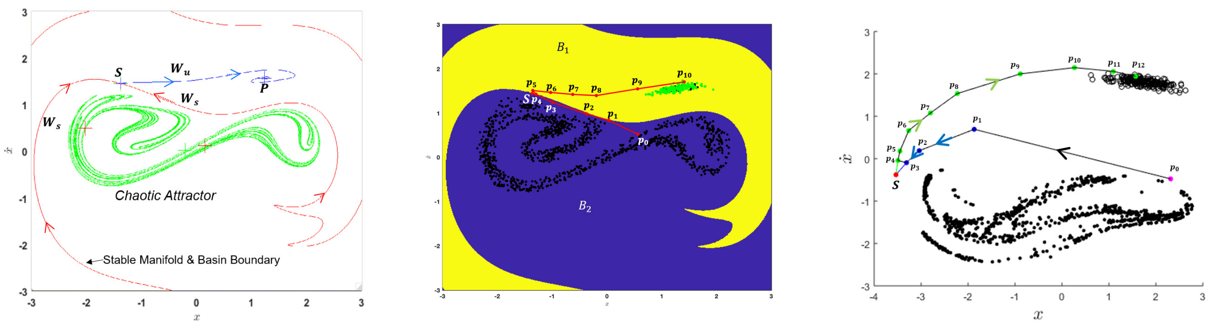
Abstract. A combination of numerical and experimental studies have been undertaken to comprehend the influence of noise on the chaotic response of bistable, Duffing oscillators. One noteworthy result of the conducted studies concerns the presence of a pair of attractors, one being periodic and the other being chaotic. It is shown that the chaotic response can be controlled and terminated with an appropriate noise level. The authors report that trajectories leave via a special escape route: the unstable manifold of a fixed point saddle on the basin boundary between the two basins of attraction.

Introduction

Dynamics of a nonlinear system can be influenced by the internal and external parameters chosen in the governing equations of motion. Bifurcation theory can be employed to understand the way in which different attractors are created or destroyed with respect to changes in the system parameters. Over the last decade, extensive results on the effect of noise on nonlinear systems dynamics have been reported (e.g., [1, 2, 3, 4]). Here, the authors subject chaotic trajectories to white Gaussian noise at low levels; that is, levels that are just sufficient to cause a trajectory to escape from the basin. It is reported that there is specific escape route that the trajectory always follows in certain systems, according to the authors' experimental and computational results. In these cases, the boundary of the basin of the chaotic attractor is the stable manifold of a periodic orbit. This manifold is captured through a stroboscopic plotting of data. As the trajectory escapes, it is observed that it is essentially on the basin boundary. The dynamics helps pull the trajectory to the fixed point on the boundary and the trajectory escapes along the unstable manifold. The experiments were conducted many times to ensure repeatability and it was confirmed that the trajectory always escapes the basin in the same way. While the present study concerns a specific system with specific parameters, the authors believe that the observed escape route would be applicable to a wide variety of nonlinear systems.



(a) Deterministic System (b) Numerical Results (c) Experimental Results

Figure 1: In plot (a), the (stroboscopic) chaotic attractor (green color) along with stable (W_s) and unstable (W_u) manifold of the fixed point saddle (point "S") for the deterministic system are shown and point "P" is a fixed point attractor. In plot (b), the numerical results for the chaotic attractor escape route (through points p_1 to p_{10}) with noise $\sigma_N = 0.02$ are shown. The green dots represent the stroboscopic map for the last 200 time periods. For plot (c), a stroboscopic map is obtained through an experimental study of a forced bistable, softening Duffing oscillator with noise amplitude $\sigma_E = 1.8$. The start of the strobe for the experiments is arbitrary and is not synchronized with the clock used for plots (a) and (b). The chaotic attractor escape route (through points p_1 to p_{12}) with noise $\sigma_E = 1.8$ is shown. The black circles represent the stroboscopic map for the last 200 time periods [3].

Results and Discussion

Numerical-experimental evidence is provided for a noise-induced escape route from a chaotic attractor to a periodic attractor. It is believed that this noise-induced escape route can be observed in a broad range of nonlinear systems. These findings also suggest that a range of noise amplitudes can be used to control the chaotic dynamics without any change in the intrinsic system parameter values.

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References

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