Classification of a family of Lorenz knots with reducible symbolic dynamics

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Abstract. Based on symbolic dynamics of Lorenz maps, we prove that, if a conjecture due to Morton is true, then a countable family of Lorenz knots associated to orbits of points in the renormalization intervals are hyperbolic. This countable family contains some of the hyperbolic Lorenz knots presented by J. Birman and I. Kofman in [2].

Introduction

Lorenz knots are the closed orbits in the Lorenz system and *Lorenz links* are finite collections of Lorenz knots. The introduction of the Lorenz template, by Williams in [9], enabled the study of Lorenz knots and links. It is a branched 2-manifold with an expanding semi-flow. Guckenheimer and Williams conjectured and later Tucker proved, that every knot and link in the Lorenz system can be projected into the Lorenz template, [8].

The *Lorenz map* is the first-return map induced by the semi-flow on the *branch line* of the Lorenz template . Periodic orbits in the semi-flow correspond to periodic orbits of the Lorenz map, so symbolic dynamics of the Lorenz map provide a codification of the Lorenz knots.

A knot is hyperbolic if its complement in S^3 is a hyperbolic 3-manifold. Thurston,[7], proved that a knot is hyperbolic *iff* it is neither a satellite knot nor a torus knot. One of the goals in the study of Lorenz knots has been their classification into *hyperbolic* and *non-hyperbolic*. Birman and Kofman listed hyperbolic Lorenz knots taken from a list of the simplest hyperbolic knots. They showed that more than half of the hyperbolic knots whose complements can be constructed from seven or fewer ideal tetrahedra are Lorenz knots, while from the 1701936 prime knots having projections with less than 17 crossings, only 20 are Lorenz.

Hugh Morton has conjectured [4],[3] that all Lorenz satellite knots are cablings (satellites where the pattern is a torus knot) on Lorenz knots. In [6], based on the work of El-Rifai, [4], we derived an algorithm to obtain Lorenz satellite braids, together with the corresponding words from symbolic dynamics.

In [5] we introduced an operation over Lorenz knots that is related with renormalization of Lorenz maps. From the point of view of symbolic dynamics, this operation corresponds to the $*$-product defined in [5] and, from [1], torus knots correspond to words that are irreducible with respect to it. So the $*$-product from symbolic dynamics of Lorenz maps is a natural way of generating non-torus Lorenz knots.

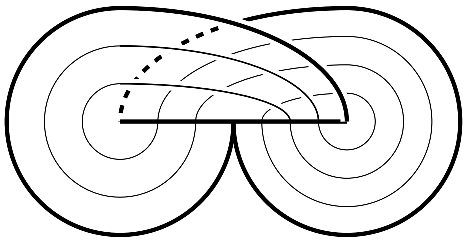

Figure 1: Lorenz torus knot with sequence $LRRLR$

Results and discussion

In this work we study an infinite family of Lorenz knots, generated through $*$-products, that contains some of the knots from Birman-Kofman's list. We prove that none of these knots is a torus knot and that, provided Morton's conjecture is true, it is not a satellite either. So, if Morton's conjecture is true, they are all hyperbolic.

References

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