An algorithm to determine the exact solution to polynomial semi-definite problems: application to structural optimization

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Abstract. A novel technique is proposed to solve polynomial semi-definite programming problems. In particular, by coupling optimization techniques with algebraic geometry tools, it is shown how to determine closed-form solutions to this class of problems. The effectiveness of the proposed technique is validated via application to some optimum truss design problems involving constraints on the global stability of the structure and on the free vibration frequencies.

Introduction

Polynomial models are ubiquitous in several applications such as: robotics [1], control systems [2], optimization [3], mathematical biology [4], game theory [5], and machine learning [6]. As an example, in [7], it has been shown that the optimum truss design problem involving constraints on the global stability of the structure and on the free vibration frequencies can be solved by determining the solution to a polynomial semi-definite (SDP) problem, that is a nonlinear program of the form

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{with} \quad M_i(x) \succeq 0, \quad i = 1,\dots,q, \quad p_j(x) = 0, \quad j = 1,\dots,k, \tag{1}$$

where $M_1(x),\dots,M_q(x)$ are polynomial mappings from $\mathbb{R}^n$ to the space of symmetric $m_i \times m_i$-dimensional matrices and $p_1(x),\dots,p_k(x)$ are given scalar polynomial functions.

The objective of this paper is to show that the joint use of optimization techniques and algebraic geometry allows one to find closed-form solutions to the optimization problem (1). In particular, the semi-definite problem (1) is first recast in a classical polynomial programming problem of the form

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{with} \quad g_i(x) \geq 0, \quad i = 1,\dots,\bar{q}, \quad p_j(x) = 0, \quad j = 1,\dots,k, \tag{2}$$

by exploiting conditions on the principal minors of the matrices $M_1(x),\dots,M_q(x)$. Secondly, algebraic geometry methods are used to determine the set of all the generalized critical points of the problem (2), among which the extrema of problem (1) lies; see Figure 1 for a graphical representation of the proposed procedure.

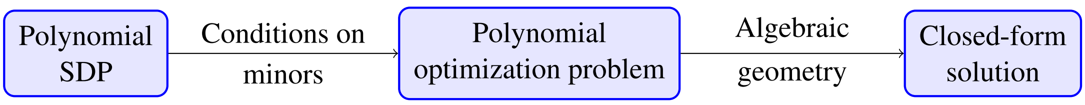

Figure 1: Graphical representation of the proposed method to solve polynomial SDPs.

Results and discussion

The main contribution of this paper is twofold. First, it is shown how to recast a polynomial SDP into a polynomial optimization problem that can be solved by using classical techniques, such as those given in [8] and [9]. Secondly, it is shown how algebraic geometry methods can be used to determine a closed-form expression for the solution to the polynomial problem. The main innovation of the procedure given herein with respect to other methods available in the literature is that it uses exact computations, thus allowing one to determine a closed-form expression for the candidate optimal values.

The proposed technique is validated and corroborated via application to some structural optimization problems involving constraints on the global stability of the structure and on the free vibration frequencies.

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