Nonparametric Identification of a Nonlinear MEMS Resonator

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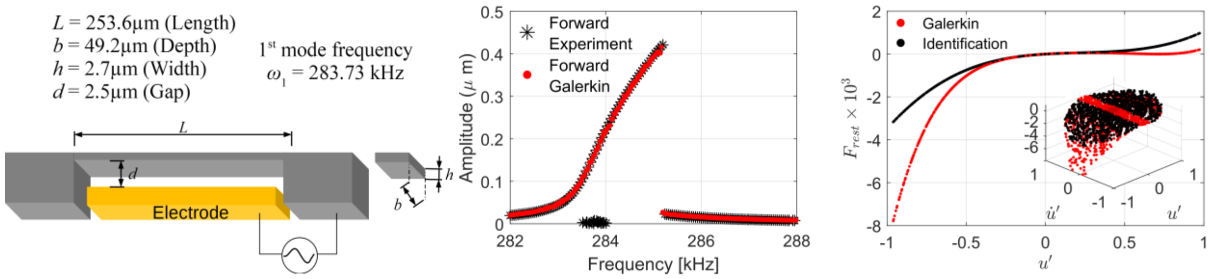
Abstract. This work presents a nonparametric identification method applied to study the nonlinear response of a MEMS resonator. The MEMS resonator is a clamped-clamped microbeam fabricated for out-of-plane motion accounting for geometric nonlinearities due to mid-plane stretching while actuated by electrostatic forces. Experimental measurements show hardening behavior in the frequency-response of the first symmetric mode of the resonator due to the dominant cubic nonlinearity. Modeling the dynamics of the microbeam using a nonparametric identification method is shown to be very effective in capturing the characteristics of the complex nonlinear system at different electrostatic AC and DC voltages.

Introduction

Accurate mathematical models bring many advantages, such as enabling predictions, and allowing the optimization of a system. Nonparametric identification methods have shown to be a valuable strategy for formulating accurate models [1-2]. These methods can obtain a function from data measurements that may provide physical representation of a system without prior knowledge of the nature of the system restoring force or nonlinearities. In addition, these techniques have been recently included to machine learning applications through neural networks and optimization algorithms [1]. This work aims to extend the application of the nonparametric identification technique demonstrated in [2] to identify the dynamic response behavior of the first symmetric mode of a nonlinear MEMS resonator through experimental data.

Results and Discussion

The MEMS device to be utilized for data measurement is a clamped-clamped beam, illustrated in Fig. 1a along with its geometric parameters and the distance between the actuation electrode and the beam. The nonparametric identification technique utilizes the state variables of a system to be identified, expressing its characteristics in terms of orthogonal functions [2]. In other words, the main purpose of the procedure is to find a restoring force using the Chebyshev polynomials. To demonstrate the method and its efficacy, the restoring force to be identified is extracted by modelling a single mode of the MEMS device considering it as an Euler-Bernoulli beam accounting for mid-plane stretching [3] through Galerkin method. Once the equation of motion is obtained, the validation with the experimental data is carried out (see Fig. 1b) for Vac = 5V and no DC load. The restoring force is obtained using the measured displacements and velocities, which must be normalized. Afterwards, a linear surface is generated from the full restoring force. Then, the selected data points are interpolated and filled with equidistant data points to improve the accuracy of the results. Using the Chebyshev orthogonality, the least-square fit of the linear surface is carried out. Fig. 1c shows the restoring force versus normalized displacement u, and the surface (on the inset) versus u and the velocity , both obtained through Galerkin (in black) and the identification method (in red). The relative error of the identified restoring force using the linear surface compared to the one obtained with Galerkin is ~48%. However, the error is very small when the displacement of the beam is -0.2 ≤ u ≤ 0.2 (~3% error). Then, using the full normalized data points to the least-square fit, good agreement is obtained. These results show that the method is very effective in capturing the nonlinear dynamical characteristics of the micro resonator.



(a) (b) (c)

Figure 1: (a) Schematic of the micro resonator. (b) Frequency-response of the microbeam, analytical simulation and experimental measurements for $V_{ac} = 5V$ and no DC load. (c) Restoring forces obtained from the Galerkin method (in black) and obtained with the identification method (in red).

References

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