Augmented perpetual manifolds of mechanical systems

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Abstract. Perpetual points (PPs) have been defined in mathematics recently and their role in the dynamics of systems is on-going research. In many unforced mechanical dissipative systems the PPs form the perpetual manifolds of rigid body motions and extending them in case of externally excited systems, lead to the definition of the augmented perpetual manifolds (APMs) of rigid body motions. In APMs, of the nonlinear and of the underlying linear system even in case that the system is excited with harmonic excitation at resonance, the displacements of all the generalised coordinates are coincide and therefore, all their velocities too. Subsequently on these motions the internal stresses of these systems are zero, which is extremely important in mechanics.

Introduction

Prasad in [1] defined the Perpetual Points (PPs), as particular points that arise by setting accelerations and jerks equal to zero for nonzero velocities. In [2] shown, that the PPs for linear unforced dissipative systems form the perpetual manifolds of rigid body motions. Examining forced systems with rigid body motions the concept of augmented perpetual manifolds (APMs) is introduced. There are several observations to be made for the significance of the mechanical system's motion in augmented perpetual manifolds and, herein the internal forces are analysed. Two, 2-degrees of freedom (dof) dissipative systems are considered,

$$1500\ddot{x} + 5 \cdot 10^5(x-y) + 3 \cdot 10^5(x-y)^3 + 346.41(\dot{x}-\dot{y}) = 5 \cdot 10^6 sin(28.87t),$$

$$1000\ddot{y} + 5 \cdot 10^5(y-x) + 3 \cdot 10^5(y-x)^3 + 346.41(\dot{y}-\dot{x}) = \frac{10}{3} \cdot 10^6 sin(28.87t),$$

and, the underlying linear one. They are excited with the linear resonant frequency. The equations of motion are numerically integrated with initial conditions $x(0) = y(0) = 1\ m$ and, $\dot{x}(0) = \dot{y}(0) = -115.47\ m/s$. In Figure (1) the responses, of both systems are depicted. The solutions are in the APMs whereas the displacements of the two masses coincide which means also their velocities. Therefore for all time instants, the internal forces, a) of the elastic potential,

$$F_{U,L} = 5 \cdot 10^5(x-y) = 5 \cdot 10^5(y-x) = F_{U,NL} = 3 \cdot 10^5(x-y)^3 = 3 \cdot 10^5(y-x)^3 = \mathbf{0}\ N,$$

and b) the damping forces,

$$F_D = 346.41(\dot{x}-\dot{y}) = 346.41(\dot{y}-\dot{x}) = \mathbf{0}\ N,$$

are all, equal to zero. It can be proved (corollary) that for mechanical systems (linear and nonlinear) admitting solutions in APMs with specific infinite types of excitation forces, the mechanical system's dynamics in APMs is internally forces-free.

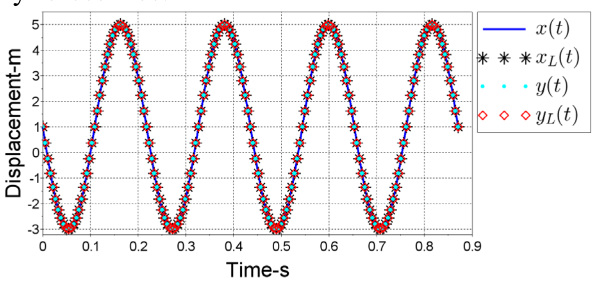

Figure 1: Displacements of the nonlinear and linear (L) system.

Results and Discussion

Two 2-dof mechanical forced systems are considered a nonlinear and the underlying linear one with particular harmonic excitation forces. The two systems although are excited by harmonic excitation at resonance, they are moving in rigid body motion whereas both masses are having the same displacement therefore, the same velocity. It is shown that, the mechanical systems on this motion are internally forces-free and this is rather significant outcome in mechanical engineering structures whereas these forces are the main cause of failures.

References

[1] Prasad A. (2015) Existence of perpetual points in nonlinear dynamical systems and its applications, *Int. J. of Bifurcation and Chaos* **25**:1530005.

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