Enriched Vlasov beam model for nonlinear dynamic analysis of thin-walled structures

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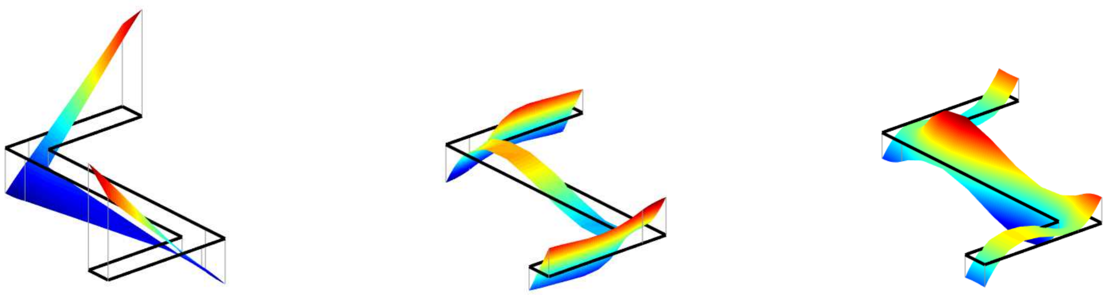
Abstract. This work presents the formulation of a three-dimensional beam finite element model with cross-section warping for the analysis of thin-walled structures under dynamic loadings. The model is defined according to a displacement approach, where generalized cross-section shear strains are independently interpolated along the element axis to avoid shear-locking. Additional warping degrees of freedom are introduced at the element nodes and cross-section out-of-plane deformations are described on the basis of Vlasov's theory, properly extended to include warping due to shear stresses. The element governing equations are derived by enforcing the stationarity of a Lagrangian functional and the element mass matrix is consistently derived. Numerical studies are presented to validate the proposed model and investigate the effects of cross-section warping on the dynamic behavior of thin-walled structures.

Introduction

Thin-walled beams are widely used in civil constructions, such as steel tall buildings and bridges, and often preferred to structural elements with compact cross-sections, as they are usually characterized by higher strength-to-weight ratios. However, nonlinear dynamic numerical analysis of thin-walled structures, often required in structural engineering studies and professional practice, is a challenging task, as behavior of such elements is strongly affected by cross-section warping and multi-axial stress interaction. Beam models are commonly used for the analysis of large scale structures because of their computational efficiency in reproducing the structural response. However, most of the beam formulations available are based on the assumption of rigid body cross-section and fail in correctly describing the response of thin-walled structures. Many enhanced formulations that include the effects of warping have been proposed in the last decades. However, only in few cases these have been developed for the analysis of large scale structures under dynamic loading conditions [1].
This work proposes an enriched three-dimensional Vlasov beam Finite Element (FE) model that includes the effect of warping due to torsional and shear actions and the inertia forces acting on the element under dynamic loading conditions. A displacement-based approach is adopted to define the element formulation, where specific degrees of freedom (DOFs) are introduced at the element end nodes and used to describe the variation of the cross-section out-of-plane deformations (Figure 1) in the element volume. Some of these DOFs are also used to independently interpolate the generalized cross-section shear strains along the beam axis, instead of the flexural rotations, so that shear-locking issues are prevented [2].
The element governing equations that include inertia effects are derived by enforcing the stationarity of a Lagrangian functional, which permits the derivation of the consistent element mass matrix [3].



(a) (b) (c)

Figure 1: Warping function shape for a S-shaped cross-section related to (a) torsion and (b-c) shear

Results and discussion

Numerical tests are conducted to validate the proposed FE model and study the effect of cross-section warping on the dynamic behavior of thin-walled structures. Modal decompositions and time-history analyses assuming nonlinear constitutive behavior are considered and the influence of in-plane cross-section deformations, that are neglected in the beam formulation, is studied. The results obtained with the proposed model are compared with those provided by enhanced beam formulations, where richer warping descriptions are adopted [3], and higher order shell or solid FE models.

References

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