Reduced-Order Modeling for Wakes around Bluff Bodies using Spectral Submanifolds

Mattia Cenedese, Andrea Massocco and George Haller

Institute for Mechanical Systems, ETH Zürich, Leonhardstrasse 21, 8092 Zürich, Switzerland

Abstract. Reduced-order modeling is among the leading theoretical challenges in the field of fluid dynamics. Several approaches exist in literature that are able to construct such models for particular classes of flows, but rarely are these approaches justified from a nonlinear dynamics perspective. In this paper, we propose an approach that identifies slow manifolds associated with steady, time-periodic or time-quasiperiodic flows and clarifies their reduced dynamics using normal form theory. Beyond establishing the mathematical framework, we present our findings in the context of vortex shedding in the wake of a cylinder.

Introduction

Despite the rise of computational capabilities, reduced-order models remain irreplaceable tools for design and control in fluid dynamics [1]. The ultimate goal is to construct a reduced-order model that can be evaluated at insignificant computational cost without compromising the precision in its results. The high-dimensional dynamical system originating from the discretization of the Navier-Stokes equations typically evolves along low-dimensional manifolds [2], hereby providing the justification for an exact reduced-order dynamical model. Dynamical phenomena arising in flows around bluff bodies received considerable attention both from theoretical and applied perspectives [2]. In the laminar regime, the flow shows an asymptotically stable steady solution which loses stability as the Reynolds number is increased. Because of a supercritical Hopf bifurcation, the wake experiences periodic vortex shedding, generating the well-known von Kármán vortex street. At larger Reynolds numbers, the periodic flow becomes unstable and the system then converges to a quasi-periodic attractor. By further increasing the Reynolds number, a fully turbulent, completely unstructured flow appears in the wake.

Results and discussion

Several approaches are present in the literature for reduced-order modeling in fluid dynamics, ranging from Galerkin projections of the Navier-Stokes equations [1] to center [3] or unstable manifold reductions [4] of the discretized Navier-Stokes equations. Data-driven or machine-learning approaches are also becoming popular [5]. A further invariant manifold approach, spectral submanifold (SSM) theory [6], is a promising candidate to tackle this problem, yet still remains unexploited in fluid dynamics. In this work, we aim to establish an exact framework for reduced-order modeling of wakes around bluff bodies. Slow manifolds attached to steady flows, limit cycles or quasiperiodic orbits can be determined as unstable manifolds, stable manifolds or eventually SSMs. Slow manifolds govern the asymptotic dynamics and lay the foundations for reduced-order models. The reduced dynamics is specially insightful when determined in its normal form. We illustrate the theoretical background and the performances of our method with a detailed discussion on the two-dimensional slow manifold that locally describes the periodic vortex shedding in the cylinder wake, illustrated in Figure 1.

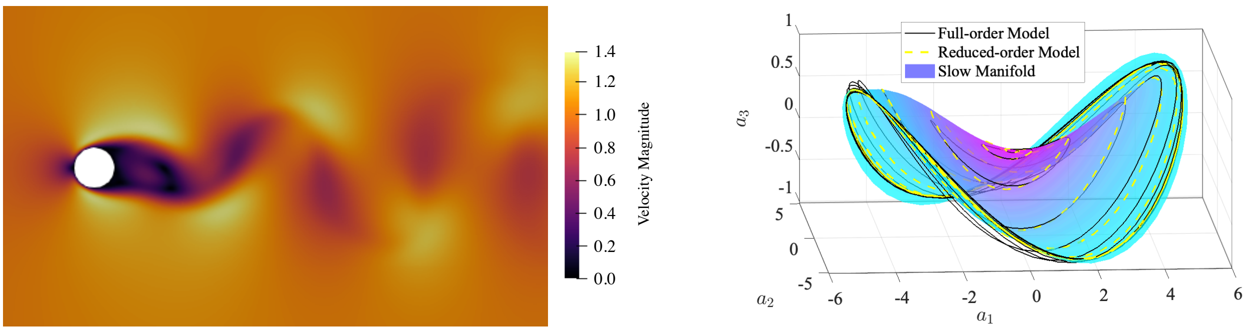

Figure 1: Left, magnitude of the velocity field of a snapshot of the full simulation at $\mathrm{Re} = 100$. Right, trajectories from full-order and reduced-order models plotted with the reconstructed slow-manifold in terms of Fourier coefficients (a_1, a_2, a_3) obtained via projection to the first three POD modes of the limit cycle.

References

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