Computation of Spectral Submanifolds and Forced Response in High-Dimensional Nonlinear Mechanical Systems

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Abstract. Finite element models of realistic nonlinear structures are characterized by very high dimensionality that renders computer simulations infeasible. Exact model reduction aims to achieve a drastic reduction of the full system unknowns in a mathematically justifiable fashion. Specifically, Spectral Submanifolds (SSM) have recently been shown to result in exact reduced-order models for periodically and quasiperiodically forced, nonlinear mechanical systems. In this work, we address challenges in the computation of invariant manifolds for realistic finite-element models and propose novel methods to counter them. We demonstrate speed gains over several orders of magnitude in obtaining forced response and backbone curves for realistic finite-element models using SSMs.

Background

The prediction of a steady-state response to an externally applied dynamic load is of special significance in engineering applications. Mechanical structures are usually characterized by light damping which results in exceedingly long integration times before a steady state is reached. Despite the broad availability of dedicated software packages [2, 3], the computation and continuation of the steady-state in response to periodic forcing remains a serious computational challenge for full scale nonlinear finite element models [4].

Recent advances in nonlinear vibrations have introduced the notion of exact model reduction via Spectral Submanifolds [1, 4], which aims to rigorously capture the essential dynamics in high-dimensional nonlinear systems and offers a reliable alternative for steady-state response predictions within feasible computation times.

Summary of results

While invariant manifolds such as Spectral Submanifolds allow for a mathematically rigorous understanding of nonlinear phenomena in dynamical systems, their applications are limited to academically simple examples in practice. Indeed, several challenges render the invariant manifold computations infeasible on engineering structures.

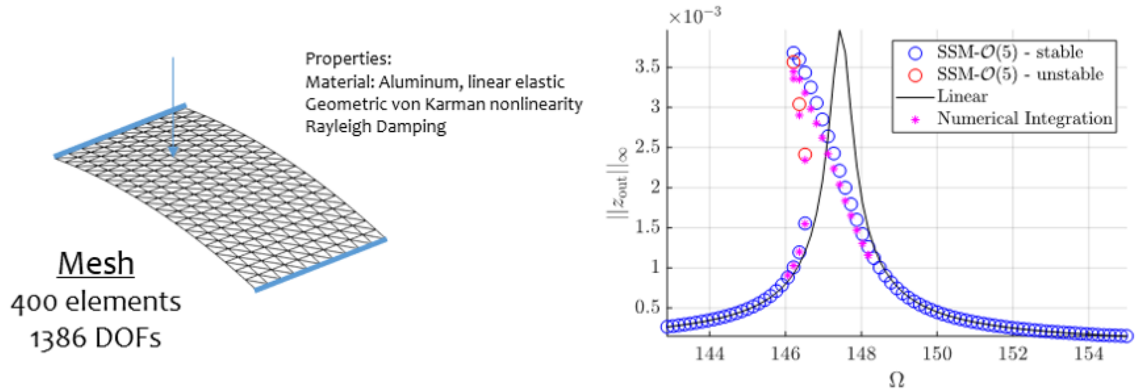

Figure 1: The forced response curve showing softening nonlinear behavior (right) for a shallow curved rectangular arch which is simply-supported on opposite ends and forced vertically, as shown (left). This forced response curve computation using a fifth-order SSM takes only 40 seconds on a standard laptop using MATLAB and shows good agreement with expensive numerical time integration used for verification purposes.

In this work, we have addressed these computational challenges and proposed novel techniques which make these computations feasible for high-dimensional nonlinear mechanical systems (see Figure 1, for instance). For the first time, this enables us to extract the forced response and backbone curves from the finite element models of realistic, geometrically nonlinear structures, such as an aircraft wing.

Furthermore, we have developed a software package that automates these computations and see it as an essential addition to the toolkit of a nonlinear modal analysis practitioner.

References

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