Semi-analytical approaches for solving Duffing oscillator with multi-frequency excitation

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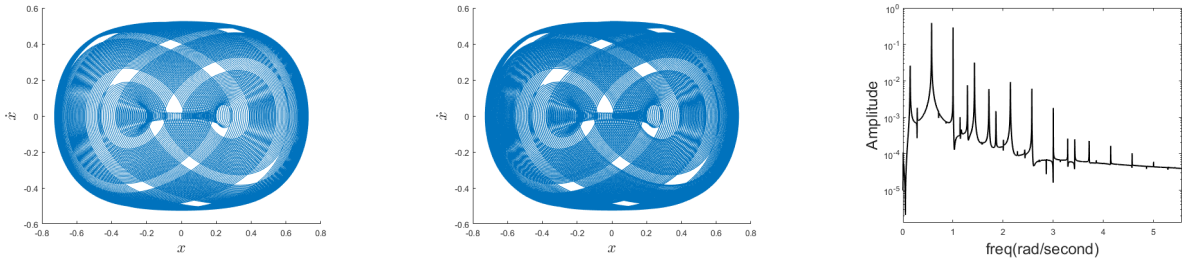
Abstract. In this paper, biharmonically excited Duffing oscillators are investigated using different semi-analytical methods. To obtain steady state response from this kind of nonlinear systems, Harmonic Balancing Method(HBM) is a well established method which is extended to Multi-Harmonic Balancing Method (MHBM) to handle systems which are quasiperiodically excited. Time Variational Method (TVM) is another recently introduced formulation which challanges the computational efficiency of HBM in case of certain systems. In this work, TVM is extended to multiple time scale formulation named as Multi-Time variational Method (MTVM). Both MHBM and MTVM are used to solve for quasiperiodically excited Duffing oscillator and the results are compared.

Introduction

A large domain of physical systems in the world can be modeled as second order dynamical system and this explains the need of its study. Duffing oscillator is one such mostly studied oscillator[1]. The Duffing equation consists of cubic term in the stiffness, which causes quasi-periodicity and chaotic behaviour in response of a simple harmonic or biharmonic excitation. Obtaining steady state solution using numerical integration is easy but not time efficient. To solve for periodic responses Harmonic Balance Method (HBM)[2] is often used, where the response is approximated by a truncated Fourier series. Multi Harmonic Balance Method (MHBM)[3, 4] is an extension of HBM where the approximated solution consists of more than one frequency corresponding to the multiple independent time scales for the system. Another method which is recently introduced by Rook[5] is Time Variational Method(TVM). Unlike HBM approach where the system is solved in frequency domain, TVM solves it in time domain itself. The reason is that in HBM, the solution is expressed in form of Fourier functions, but in TVM it is expressed in form of basis functions which are distributed in time domain. This reduces the effort and computation of AFT (Alternate Time Frequency) process in each iteration. But till now TVM is not extended to predict solutions containing multiple time scales. In this paper, a MHBM equivalent multi-frequency time variational technique (MTVM) is formulated. MTVM along with MHBM is used for obtaining phase portrait and frequency responses of quasiperiodically excited Duffing oscillator.



(a) Response obtained from MHBM (b) Response obtained from MTVM (c) Frequency domain plot

Figure 1: Response of a Duffing oscillator under quasi-periodic excitation

Result

The response of quasi-periodically excited Duffing oscillator is shown in figure 1. In these kind of systems, the number of iteration is more in MTVM than MHBM, but the computation time per iteration is less for MTVM, which makes overall elapsed time lesser than MHBM. This methods are further used to find frequency response of a typical Duffing system. Here multiple solution and bifurcation domain is obtained using continuation algorithm by varying one of the forcing amplitudes in the quasiperiodic excitation.

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