Perturbations for non-local elastic vibration of circular arches

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Abstract. Circular arches are curved beams with usual kinematical assumptions. The finite non-local elastic field equations are perturbed in terms of two scalar parameters: a) an evolution one, providing how the actual and the reference shape are detached; b) a material one, providing the fraction of non-local response. Thus, we get hierarchies of equations describing the effect of: i) geometrical non-linearities on kinematics and balance; ii) the long-range interaction on static and dynamic response. Pattern solutions for the lowest levels of these hierarchies are found and discussed.

Introduction

We consider plane shallow circular arches (used, e.g., in MEMS and NEMS) and model them as curved beams, i.e., deformable lines to which plane figures (the cross-sections) following coarse kinematics are attached. We study how: a) the non-linear static response to a uniform load affects linear vibration; b) a portion of a non-local elastic material influences the response. We consider rigid cross-sections, finite kinematics, and balance in the actual shape. We perturb these equations (in non-dimensional form) in terms of a scalar describing the evolution of the arch shape. In slender arches it is assumed, and confirmed by experience, that shearing is negligible. Experience, numerical results of literature, and our previous work [1] suggest the relative orders of magnitude of the geometrical quantities, so we limit the perturbation expansion to a suitable order.

The balance equations are coupled and account for the small non-linear contributions of the non-zero strains and non-infinitesimal, moderate rotation of the cross-sections. The material response is linear and decoupled, consisting of local (short-range) and non-local (long-range) fractions that obey Eringen’s integral model: stress depends on strain via a constant plus the convolution integral of a kernel function [2]. Such law, being superposed to a perturbed system of kinematic and balance equations, provides an integral-differential system of field equations that is tackled by another perturbation in terms of the scalar fraction of non-local response [3].

Results and Discussion

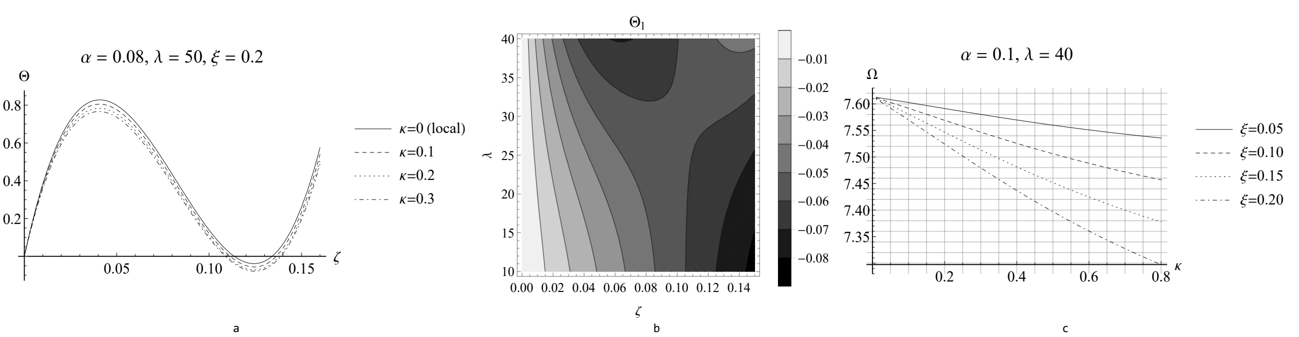

Figure 1: Load vs. displacement (a); decrement of the load due to non-locality (b); frequency of the 1st symmetric mode (c).

Numerical solutions to the first sets of the hierarchies of problems are found as in [1]. Fig. 1(a) shows that in an arch with shallowness $\alpha = 0.08$ and slenderness $\lambda = 50$ (hence, very shallow and slender), with a volume fraction of non-local material $\xi = 0.2$, the load vs. crown displacement is a curve exhibiting possible snapping; the influence of the non-local parameter κ is limted, yet non negligible, and does not change the qualitative behaviour of the arch. Fig. 1(b) shows that in a displacement controlled test the non-locality and the slenderness act as to decrement the applied load in a limited but appreciable way. Fig. 1(c) shows how the volume fraction of non-local material and the non-locality parameter lower the non-dimensional first natural angular frequency, as physically expected.

References

[1] Eroglu U., Ruta G. (2020) Vibration of pre-loaded shallow circular arches, in: Nonlinear Dynamics of Structures, Systems and Devices 237-245, Cham, Springer.

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[3] U. Eroglu. (2020) Perturbation approach to Eringen’s local/non-local constitutive equation with applications to 1-D structures. *Meccanica* **55**:1119–1134.