Nonlinear dynamics of a hyperelastic cylindrical shell composed of the incompressible Ogden material

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Abstract. In this paper, nonlinear dynamic behaviours are examined for a hyperelastic cylindrical shell composed of the incompressible Ogden material, where the shell is subjected to uniform radial periodic loads at the internal surface. A second order nonlinear differential equation describing the radial symmetric motion of the shell is obtained. Then, the dynamic characteristics of the system are qualitatively analyzed in terms of different values of material parameters (including power rate types) and structure parameter. Particularly, for the autonomous system, the number of equilibrium points and some interesting characteristics of phase diagrams are determined. It is shown that there exist critical loads, the phase diagrams may be the asymmetric homoclinic orbits. For the nonautonomous system, it is proved that near the centre point, the level curves break into islands and the shell performs a quasi-periodic motion; near the saddle point, the shell performs a chaotic motion.

Introduction

Axisymmetric structures composed of hyperelastic materials have important applications in many engineering fields. The dynamic problems of related structures have attracted much attentions in which the originated work was presented by Knowles [1]. Then, the related studies have been further extended. Recently, Aranda-iglesias [2] investigated the large amplitude free vibrations of a hyperelastic cylindrical structure. Zhao [3] studied the dynamic behaviours of a hyperelastic spherical film under the periodic disturbance in detail. More literatures on related problems can be seen in the review [4] and the monograph [5], and so on. In this paper, the radial symmetric motion of a cylindrical shell composed of the incompressible Ogden materials are examined. Firstly, a second order nonlinear ordinary differential equation is established, as follows,

$$x\ddot{x}\ln\left(\frac{\delta+x^2}{x^2}\right)+\dot{x}^2\ln\left(\frac{\delta+x^2}{x^2}\right)-\dot{x}^2\frac{\delta}{\delta+x^2}+\frac{2}{\rho R_1^2}\int_{\left(\frac{\delta+x^2}{1+\delta}\right)^{1/2}}^{x}\frac{\mu_1(-k^{-\alpha_1-1}+k^{\alpha_1-1})+\mu_2(-k^{-\alpha_2-1}+k^{\alpha_2-1})}{k^2-1}dk=\frac{2}{\rho R_1^2}p(t)$$

Then, the important influences of material parameters and structure parameter on the nonlinear dynamic characteristics are discussed. Some numerical examples are shown in Fig. 1.

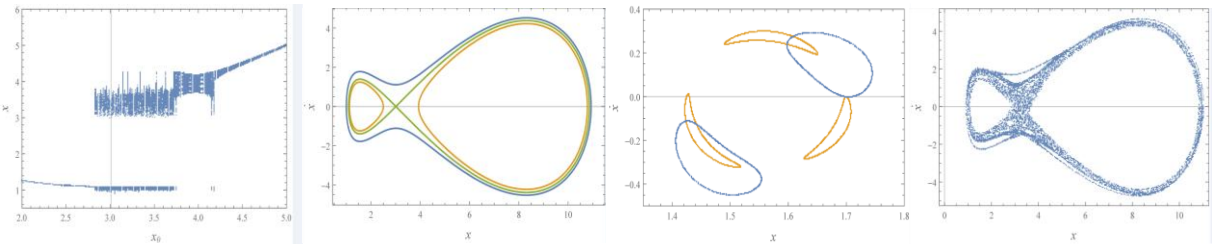

Figure 1: Bifurcation diagram, phase orbits with the asymmetric homoclinic orbit, the Poincaré sections of quasi-periodic, chaotic motions

Results and discussion

Some of the specific results are as follows, i) the material parameters have important influences on the number of the equilibrium points, correspondingly, the qualitative properties of the system are completely different, sometimes there are asymmetric homoclinic orbits; ii) Under the periodic perturbation, if the initial conditions are near the centre point, both the systems perform quasi-periodic oscillation. However, if the initial conditions are near the saddle point, the system with two equilibrium points diverges rapidly with the perturbation, which finally leads to the structural damage; while the system with three equilibrium points performs chaotic motion, and the structure will not be destroyed.

References

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