Fixed-interval smoothing of an aeroelastic airfoil model in incompressible flow

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Abstract. Fixed-interval smoothing, as one of the most important types of state estimation, has been considered in many practical engineering especially in the analysis of flight test data. In this study, a novel algorithm is introduced to explore the fixed-interval smoothing problem of a conceptual two-dimensional airfoil model in incompressible flow from noisy measurement data. A single objective optimization problem is constructed with the classical Runge-Kutta scheme, and then estimations of the system states, the measurement noise and even the unknown parameters can be obtained simultaneously through optimizing the objective function. Finally, its effectiveness and feasibility are examined through several simulation results.

Introduction

Over the past few decades, nonlinear dynamics of the aeroelastic airoil models have attracted widespread attention [1, 2, 3, 4, 5]. Fixed-interval smoothing is to estimate the system state processing all of the given measurements in a fixed time interval, which is the most important type for practical engineering applications. For example, estimation of aircraft motions along a flight trajectory or accident analysis of the aircraft from the flight records with noisy fluctuation can be formulated as a fixed-interval smoothing problem. Therefore, only the fixed-interval smoothing estimation is the interest of the present study.

Results and Discussion

In general, governing equation of the airfoil models can be rewritten in

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}\right), \tag{1}$$

in which $\boldsymbol{x}$ denotes the state vector, $\boldsymbol{f}\left(\boldsymbol{x}\right)$ is the vector field. The noisy measurement data is generated as

$$\boldsymbol{Y} = \boldsymbol{X} + \text{noise\_level} \cdot \text{std}\left(\boldsymbol{X}\right) \cdot \boldsymbol{\epsilon}_{\text{noise}},$$

here $\boldsymbol{X} = \{\boldsymbol{x}_j\}_{j=1}^{N}$ is the sampling data from true trajectory simulated by the Eq. (1), $\boldsymbol{\epsilon}_{\text{noise}}$ represents the measurement noise, noise_level is the level of measurement noise in percent, and std $\left(\boldsymbol{X}\right)$ denotes a diagonal matrix and its diagonal elements consist of the standard deviation of each state. Several simulations for different cases are implemented to show effectiveness of the presented scheme. As an example, Fig. 1 shows measurement state, true state, estimation state and measurement noise for the pitch motion of the airfoil system, in which the measurement data is corrupted by 40% Gaussian white measurement noise. All the obtained results indicate that the introduced algorithm can achieve accurate state and parameter estimations for the airfoil model with different measurement noises, and still available for high-dimensional systems.

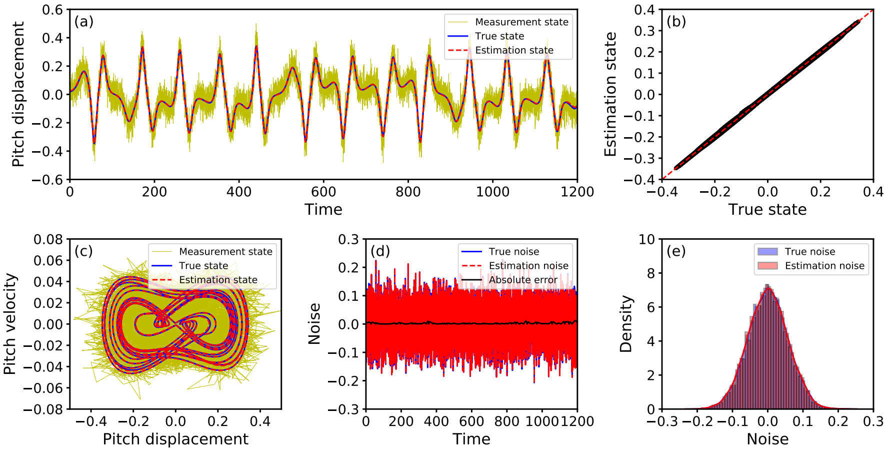

Figure 1: Estimation results of the pitch motion for the airfoil model with cubic structural nonlinearity and without unknown parameters in the chaotic regime, corrupted by 40% Gaussian white measurement noise with zero mean. (a) time history of pitch motion; (b) relationship between true and estimation state; (c) phase diagram of pitch motion; (d) time history of measurement noise; (d) density histogram of the measurement noise.

References

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