Invariant spectral foliations for model order reduction and synthesis

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Abstract. In this talk we introduce a method that distils a reduced order model from time-series data. To this end an invariant spectral foliation (ISF) is used, which is the smoothest and therefore unique invariant foliation about an equilibrium of a dynamical system tangent to an invariant linear subspace. We show that an ISF can be directly identified from data and that backbone and damping curves can be calculated. Multiple ISF can also be used to fully reconstruct the dynamics. Therefore the method is a natural nonlinear generalisation of linear modal analysis, testing and synthesis.

Introduction

This abstract introduces [6]. An invariant foliation is a decomposition of the state space of a dynamical system into a family of manifolds, called leaves, such that the dynamics brings each leaf into another. If a leaf is brought into itself, then it is also an invariant manifold. A foliation is generally characterised by its co-dimension, which equals the number of parameters needed to describe the family of leaves so that it covers the state space. The reduced order model (ROM) is the dynamics that maps one leaf of an invariant foliation into another leaf and has the same dimensionality as the co-dimension of the foliation. Such ROM treats all initial conditions within one leaf equivalent to each other and characterises the dynamics of the whole system. Therefore the ROM is imprecise about the evolution, it can only tell which leaves a trajectory goes through. This also means that the ROM applies to all initial conditions of the original system, in contrast to invariant manifolds, where the initial condition must come from a lwo dimensional surface to have a valid prediction.

Multiple ISFs can act as a coordinate system about the equilibrium. When individual leaves from different foliations intersect in one point, the dynamics can be fully reconstructed from the foliations. Therefore ISFs are fully paralleled with linear modal analysis of mechanical systems [1]: it allows both the decomposition of the system and the reconstruction of the full dynamics. To reconstruct the dynamics one needs to find intersection points of leaves from different foliations which is more complicated than adding vibration modes of linear system. However, such composability is not at all possible with invariant manifolds or any other nonlinear normal mode (NNM) definition [2, 3, 4, 5]. Therefore, an invariant foliation seems to be the closest nonlinear alternative of linear modal analysis. The concept of composition is illustrated in figure 1.

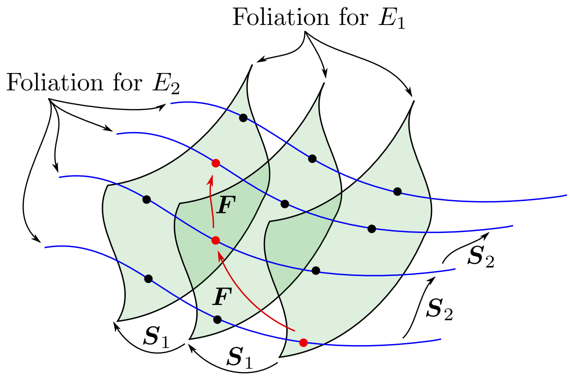

Figure 1: Two foliations act as a coordinate system. An initial condition (red dots) is mapped forward by $\boldsymbol{F}$, however each leaf of a foliation is brought forward by the lower dimensional maps $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$. Due to invariance of the foliation, the full trajectory can be reconstructed from the two maps $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ and the leaves of the foliations.

References

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