Stability analysis for a class of non-stationary impulsive switched systems

Alexey Platonov*

**Saint Petersburg State University, St. Petersburg, Russia*

Abstract. In the paper, the stability problem is investigated for some classes of differential systems under the influence of switching and impulsive actions. The Lyapunov direct method is used. It is assumed that Lyapunov function constructed for the considered system satisfies the non-stationary differential inequality with hybrid degrees. Especially interesting for analysis is the case where non-stationary coefficients in this differential inequality disappear with time, or, on the contrary, increase unboundedly. Some examples of dynamical systems, for which one can apply the obtained results, are considered. So, hybrid complex systems with linear and nonlinear subsystems are studied. Also, a class of nonlinear mechanical systems with non-stationary switched force fields is investigated.

Introduction

The theory of switched and impulsive systems has been actively developed over the past decades. Such systems have many important practical applications. Switching usually involves changing the structure of the considered system while maintaining the continuity of its solutions. Impulsive effects lead to an instantaneous jumps of the solutions. Switching and impulsive effects can be caused by external influences. They can also be a part of a control design. One of the most important problems associated with impulsive switched systems is the stability problem. The combination of the direct Lyapunov method with the theory of differential inequalities is the basis for most studies on the stability of hybrid systems. However, most of the studies involved only linear differential inequalities for Lyapunov functions, which were used to establish conditions for exponential stability. Meanwhile, many practical processes can be modeled by systems with essentially nonlinear dynamic behaviors, and exponential stability may not be inherent to such systems. The presence of non-stationary parameters in the considered system can lead to fundamentally new effects in comparison with autonomous systems. Continuous changes of parameters in the impulsive switched system are superimposed on discrete changes of operating modes and impulse actions, and, as a result, many characteristics, for example, the value of dwell time, can get a non-stationary (changeable) character. It is worth noting that systems with discontinuous non-stationary coefficients are quite typical for many practical processes. Such systems can be also considered as switched systems with, generally, infinite numbers of operating modes. Often, the behavior of non-stationary parameters changes significantly, and as a result, different estimates for used Lyapunov functions are constructed at different time intervals. In addition, Lyapunov functions usually also depend on the system parameters. Therefore, the discontinuity of these parameters leads to the need to use discontinuous (multiple) Lyapunov functions. As a rule, researchers try to limit non-stationary parameters by some constants in order to deal with autonomous estimates. However, this approach does not work if the coefficients change their sign, are unbounded, or, on the contrary, tend to zero. The problem also arises of evaluating certain non-stationary expressions to obtain the required differential inequalities for Lyapunov functions.

Results and discussion

In the present paper, we assume that a multiple Lyapunov function satisfying the some non-stationary differential inequality with hybrid degrees is constructed. The change of degrees in the differential inequality can be caused either by a change of operating modes or by a change in the behavior of non-stationary coefficients in the considered system. We use both linear and nonlinear differential inequalities, i.e. we suppose that the system can describe both linear and nonlinear dynamics at different time intervals. The main purpose of the paper is to find asymptotic stability conditions for solutions of the investigated impulsive switched system. Also, we consider some special classes of systems for which the obtained results are applicable. So, the stability problem for non-stationary complex system is studied. We assume that among the interacting subsystems there are both linear and nonlinear. Also, nonlinear mechanical system under the influence of non-stationary switched dissipative and potential forces is considered.

The present paper is a development of the results obtained in [1, 2].

Article Highlights: i) the influence of the combination of such factors as nonlinearity, non-stationarity, switching and impulsive effects on the stability of dynamical systems is studied; ii) new asymptotic stability conditions for hybrid systems are obtained; iii) the results are applied to the analysis of some classes of complex and mechanical systems.

References

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