# Nonlinear drill-string vibrations with random distributed unbalance

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**Abstract.** In this paper, a deterministic lateral-torsional drill-strings model is provided with distributed unbalance. In order to analyse the effects of the distributed unbalance, a second model is developed for random distributed unbalance. Finally, the model is used in several drilling configurations to evaluate the presence of safe work zones.

## Introduction

In the oil and gas industry, drill-strings are used to drill rock-formations until the reservoir is reached. They are usually divided in two main components: (i) the drill-pipes and (ii) the Bottom Hole Assembly (or BHA). The former is extremely slender and can reach up to some kilometres of extension. The latter is located at the bottom and is where most of the equipment is located. Whereas lateral vibrations originated from the unbalance are mostly harmless, in extreme cases the impact between the drill-string and the borehole wall may lead to a severe phenomenon known as backward whirl. On the other hand, the bit-rock interaction can lead to a severe torsional dynamic: the stick-slip. The relations between lateral and torsional dynamics are often of nonlinear nature and hence, one severe dynamic may directly affect the other, i.e.: backward whirl may lead severe torsional vibrations and the severe torsional vibrations may lead to backward whirl. In this work, a continuous model is provided and discretized with the Finite Element Method. This model takes into account static axial forces [1] and considers a distributed unbalance [2] in the BHA, which is seldom applied to drill-string models:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F_{bit}}(\dot{\mathbf{u}}) + \mathbf{F_{bh}}(\mathbf{u}, \dot{\mathbf{u}}) + \mathbf{F_{un}}(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the generalized inertia, damping and stiffness matrices. $\mathbf{u}$ is the system‘s generalized displacements. The generalized nonlinear forces $\mathbf{F_{bit}}(\dot{\mathbf{u}})$, $\mathbf{F_{bh}}(\mathbf{u}, \dot{\mathbf{u}})$ and $\mathbf{F_{un}}(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}})$ refers to the bit-rock interaction, the discontinuous contact forces and the unbalance force, respectively. Finally, a stochastic model is developed where the phase angles between unbalanced BHA sections ($\alpha_1$ and $\alpha_2$ ) are modelled as random variables. With the model developed, a study concerning the effects of the unbalance is conducted.

## Results and Discussion

Each time-domain integration was conducted with the central difference method, where the time-step integration was small enough to treat accurately the system's discontinuities. The system presented a high dependence on the BHA section's unbalance angle. When comparing the unaligned ($\alpha_1 = \alpha_2 + \pi/2$) and aligned ($\alpha_1 = \alpha_2 = 0$) unbalance distributions, it can be noticed that the phase angles can induce severe torsional vibrations. In summary, if an unbalance profile can be set arbitrary, it can be chosen to mitigate not only lateral vibrations, but also torsional vibrations.

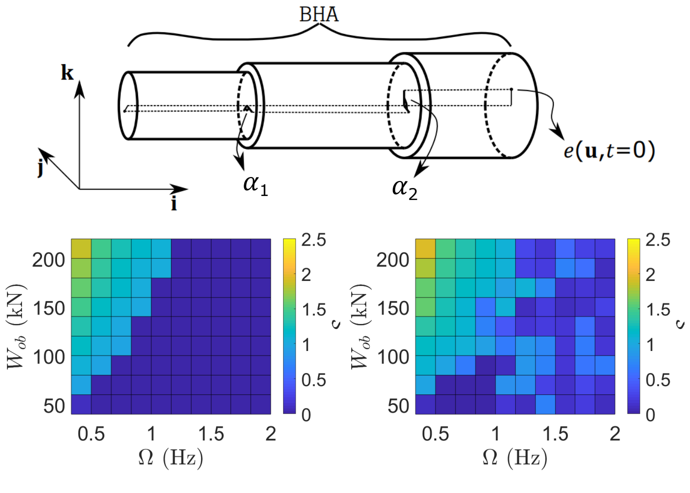

Figure 1: Sketch of BHA unbalanced sections and torsional severity maps with unaligned and aligned unbalances, respectively.

## References

[1] Khulief Y. A., and Al-Naser, H. (2005) Finite element dynamic analysis of drillstrings. Finite Elements in Analysis and Design, 41(13), 1270–1288. https://doi.org/10.1016/j.finel.2005.02.003 .

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