Adapted SIQR model for the dynamics of SARS-CoV-2 infection

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Abstract. This paper adapts the Susceptible-Infected-Quarantined-Recovered model (aSIQR) for describing the dynamics of the SARS-CoV-2 virus propagation, responsible for the COVID-19 pandemic. The model assumes a piecewise constant transmission rate, accounting for the effects of lock-downs in affected countries. Numerical results are shown for Portugal (PT), United States (US), and France (FR). There is a good agreement of the simulations and the data available for these countries.

Introduction

December 2019 was the dawn of the COVID-19 pandemic, at Wuhan, China. Severe consequences in terms of human lives, and healthcare, social and economical systems, are seen worldwide. Increased values of infected and dead people are still observed, being the three major contributors the United States, Brazil and India [1]. Epidemiologists are constructing short- and long-term projections to prepare for, and potentially mitigate, the spread and impact of SARS-CoV-2. Nevertheless, the future depends on a number of unknowns: lasting immunity, effects of seasonality on virus spread, and government and individual's choices. We develop a nonlinear differential model and discuss the results when fitting to real data from COVID-19 pandemic in PT, US and FR.

The proposed model

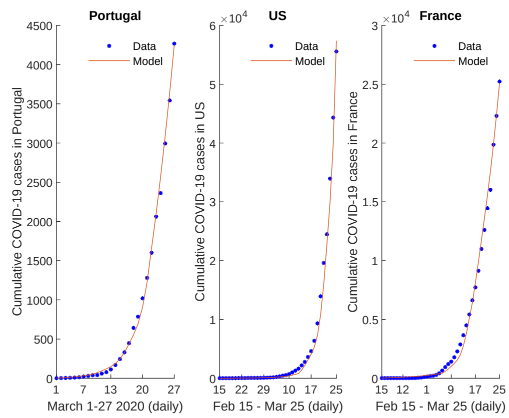
Let t be time (days), the total population is divided in eight classes, namely: $S(t),\ I_c(t),\ I_u(t),\ Q_h(t), H(t),\ R(t), R_m(t)$ and $D(t)$, which denote the number of susceptible, infectious showing symptoms, infectious asymptomatic, hospitalized, quarantined at home, recovered, and recovered from asymptomatic class individuals, and the number of deaths attributed to the infection, respectively. Thus, the total population is given by $N(t) = S(t)+I_s(t)+I_u(t)+Q_h(t)+H(t)+R(t)+R_m(t) + D(t)$, and it is assumed constant. In order to take into account the effects of national lock-downs on epidemic dynamics, a piece-wise parameter β is herein introduced following [2]. It is assumed that β varies according to $\beta = \beta_0$ (initial rate), for $t < T_1\tau_1$, $\beta = \beta_1 = \beta_0\rho_1$, for $T_1 + \tau_1 < t < T_2 + \tau_2$, $\beta = \beta_1\rho_2$, for $t > T_2 + \tau_2$, where $\rho_i \in \mathbb{R}$, and τ_i, $i = 1, 2$, account for the delays in the appearance of measures' effects in the pandemic's dynamics. The nonlinear system of ODEs describing the dynamics of the aSIQR model is given by:

$$
\begin{aligned}
\dot{S}(t) &= b - \frac{\beta}{N} S(t)\left(\xi I_s(t) + I_u(t)\right) - dS(t) \\
\dot{I}_s(t) &= \frac{\beta(1-p)}{N} S(t-\tau)\left(\xi I_s(t-\tau) + I_u(t-\tau)\right) - \\
&\quad - I_s(t) - dI_s(r) \\
\dot{I}_u(t) &= \frac{\beta p}{N} S(t-\tau)\left(\xi I_s(t-\tau) + I_u(t-\tau)\right) - \\
&\quad - (\epsilon + \mu)\, I_u(t) - dI_u(t) \\
\dot{Q}_h(t) &= qI_s(t) - (\lambda + \mu_q)\, Q_h(t) - dQ_h(t) \\
\dot{H}(t) &= (1-q)I_s(t) - (\lambda + \mu_h)\, H(t) - dH(t) \\
\dot{R}(t) &= \lambda\left(Q_h(t) + H\right) - dR(t) \\
\dot{R}_m(t) &= \epsilon I_u(t) - dR_m(t) \\
\dot{D}(t) &= \mu_q Q_h(t) + \mu_h H(t))
\end{aligned}
\qquad (1)
$$

Parameter of model (1)	Description
b	birth rate
d	natural death rate
β	contact rate
ξ	modification parameter
p	rate of asymptomatic (unknown) infectious
ϵ	recovery rate of asymptomatic infectious
$\mu_q,$	disease mortality rate for quarentined at home
μ_h	disease mortality rate for hospitalized individuals
q	rate of hospitalized infected individuals
λ	recovery rate of hospitalized infected individuals

Discussion and conclusions

The proposed aSIQR model provides good fits to real data and clarifies the contribution of lock-downs in mitigating COVID-19 pandemic. PT has seen an early implementation of the lock-down, as opposed to US and FR. The cumulative number of infectious people in these countries suggests it was an adequate health policy. This work will be extended to other countries and regions.



References

[1] The Guardian `https://www.theguardian.com/world/2020/aug/05/coronavirus-world-map-which-countries-have-the-most-covid-19-cases-and-deaths`, assessed August 06 2020.

[2] Gaeta, G. A simple SIR model with a large set of asymptomatic infectives. arXiv preprint 2003.08720, (2020). doi: 10.3934/mine.2021013.