Self-excited Oscillations with Two different Mechanisms in A Piecewise Smooth nonlinear Rotor/Stator Rubbing System

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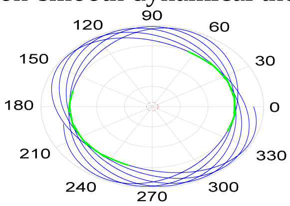
Abstract. In this work, a piecewise smooth nonlinear rotor/stator rubbing system is investigated with the focus on unveiling the characteristics of two kinds of self-excited oscillations due to dry friction and cross-coupling stiffness. Also the characteristics of stick-slip oscillations exhibited in the self-excited dry friction backward whirl (DFBW) is explored from a point of view of non-smooth sliding bifurcations based on the Filippov's convex method. The methodology employed and the results obtained may provide deeper insight into the dynamical characteristics of the piecewise rotor/stator rubbing systems and the potential guidance for the design of rotor systems.

Introduction

The dynamics of engineering systems, which are generally governed by nonlinear, sometimes also non-smooth, differential equations, are very complicated. Rotor-to-stator rubbing, which is usually modeled by multiple degree-of-freedom non-smooth nonlinear systems, is one of such examples and of great practical interests, i.e., for the safety of operation of turbomachinery. Actually, two instability mechanisms may appear simultaneously during rotor-to-stator rubbing, namely, dry friction effect and cross-coupling effect, which might induce a self-excited dry friction backward whirl or a self-excited forward whirling motion of the rotor system. The former one possesses the stick-slip transitions that can be characterized by sliding bifurcations in the non-smooth dynamical theory.



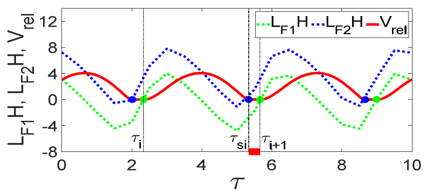


Figure 1: The dry friction backward whirls with stick-slip oscillations. (left) Rotor orbit in pure rolling (green curve) and in slipping (blue curve); (right) Time histories of relative velocity and normal components of vector fields.

The Piecewise Smooth Rotor/stator Rubbing System

The non-dimensional equations of motion in polar coordinate system for the rotor/stator system under dry friction backward whirl can be written as,

$$\begin{aligned} &R'' + 2\zeta R' + (\beta+1)R - \phi'^2 R - R_0 = \Omega^2 \cos(\Omega\tau - \phi) \\ &R\phi'' + 2\zeta R\phi' + 2R'\phi' + F_\mu = \Omega^2 \sin(\Omega\tau - \phi) \\ &V_{\rm rel} = \Omega R_{\rm d} + \phi' R \end{aligned} \quad (1)$$

with Coulomb friction defined as: $F_\mu = \mu(R - R_0)\cdot \mathrm{sgn}(V_{\rm rel})$. Let $x_1 = R$, $x_2 = R'$, $x_3 = \phi$, $x_4 = \phi'$ and $\theta = \Omega\tau \bmod 2\pi$. Eq. (1) is recast into first order autonomous ODEs with discontinuous right-hand side.

$$x' = \begin{cases} F_1(x) & \text{for} \quad H(x) > 0 \\ F_2(x) & \text{for} \quad H(x) < 0 \end{cases} \qquad x = (x_1, x_2, x_3, x_4, \theta)^{\rm T} \in \mathbb{R}^5 \quad (2)$$

where the *switching hypersurface* is dfined as: $\Sigma := \{x \in \mathbb{R}^5 : H(x) = \Omega R_{\rm d} + x_1 x_4 = 0\}$. Since $F_1 \neq F_2$ at $x \in \Sigma$, the system given by Eq. (2) is said to be a system of Filippov type with one degree of smoothness.

The sliding regions Σ_s and its boundaries in the switching manifold are given by

$$\Sigma_s = \{x \in \Sigma :\ \mathcal{L}_{F_1} H(x) \cdot \mathcal{L}_{F_2} H(x) \le 0\} \quad \text{and} \quad \partial\Sigma_s = \{x \in \Sigma :\ \mathcal{L}_{F_1} H(x) \cdot \mathcal{L}_{F_2} H(x) = 0\} \quad (3)$$

where $\mathcal{L}_F$ denotes Lie derivative $\mathcal{L}_F H(x) = \dfrac{\partial H(x)}{\partial x} F$ along the flow of F and $\partial H(x)/\partial x$ is the gradient of $H(x)$.

Results and Discussion

The interplay between cross-coupling stiffness and dry friction on self-excited forward whirling motion of the rotor/stator rubbing system, which is directly related to stable NNMs, is demonstrated. Three types of sliding regions on the switching manifold, which show the characters of stick-slip transitions in DFBWs, are identified according to the conditions (as shown in Fig.1). The influence of different parameters on the sliding bifurcations and the corresponding boundaries is explored. Some experimentally observed phenomena in DFBWs can be confirmed and well explained through the current analysis.

References

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