State-dependent switching law for stabilization to a switched time-delay system with two unstable subsystems

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Abstract. A method for designing a state-dependent switching law to stabilize a class of switched nonlinear systems with two unstable delayed subsystems is presented. With a small delay, the energy change of each subsystem can be analyzed by using the equivalent stiffness and negative damping based on second-order Taylor approximation. Then, two approximated optimal switching curves that maximize the energy loss at switching are obtained by introducing a performance index functional and using the variational principle. Both stable equilibria and periodic solutions can be produced at different delay values.

Introduction

A switched system is composed of a family of subsystems with a switching rule between the subsystems, and a switched system with all unstable subsystems can be stabilized by a properly designed switching rule [1]. However, as in the spacecraft attitude fault tolerant control, stability and dynamics analysis of switched nonlinear delayed systems with unstable subsystems is challenging [2]. For switched systems with time delays, where the delays come from controllers and filters as in control applications for example, Lyapunov-Krasovskii functionals and dwell time methods are major tools for stability analysis of systems with time-dependent switching laws [3]. The obtained results depend on estimation techniques and thus might be conservative. In addition, the stability is in the sense of asymptotic stability, and it may take a long time before the switched system is stabilized. In this abstract, a robust switching law is designed to have less conservative results and approximately optimized decaying ratio of the solution.

Method

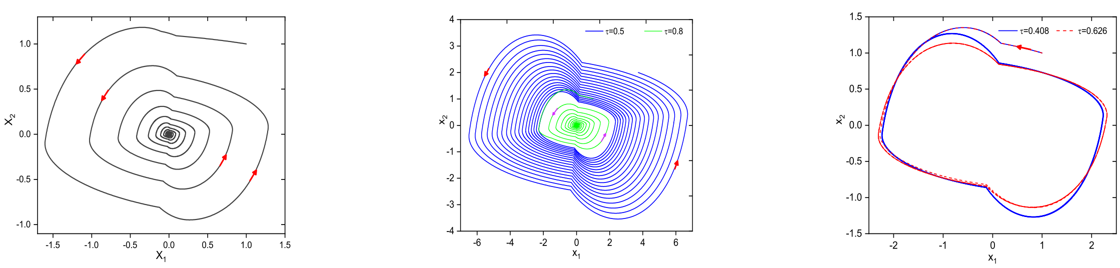
Let us consider a switched time-delay system with two unstable subsystems

$$\begin{cases} \dot{x}_1 = f_i(x_1, x_2) \\ \dot{x}_2 = g_i(x_1, x_2, x_1(t-\tau)), \qquad (i = 1, 2), \end{cases} \tag{1}$$

where $\tau \geq 0$ is the time delay, f_i, g_i are smooth nonlinear functions, and $f_i(x_1, x_2)$ is monotonous with respect to x_2. The switching law is assumed pure state-dependent: $\psi(x_1(t), x_2(t)) = 1$ or 2 when $(x_2 - \phi_1(x_1))(x_2 - \phi_2(x_1)) \geq 0$ or < 0 respectively. Switched mechanical systems with a delay are focused. Assume that the delay is short, then the delay term can be approximated by the second-order Taylor expansion. Based on the energy function of each approximated subsystem, a performance index depending on a switching curve $x_2 = \phi_(x_1)$ is introduced. Two switching curves are determined when the system energy loss at switching are maximized and minimized. The stability of steady states and identification of periodic solutions of the switched delay system can be carried out in a simple procedure.

Result

With $f_1 = x_1 - 4x_2,\ f_2 = x_1 - 16x_2,\ g_1 = 4x_1 + x_2 + x_1(t-\tau),\ g_2 = x_1 + x_2 + 0.25x_1(t-\tau)$, for example, the trajectory approaches to the origin in a spiral-like way at $\tau = 0.1$ as shown in Fig.1(a), at a fast speed due to approximated optimized switching curves. When $\tau = 0.5$, the switched system is unstable, and stable again at $\tau = 0.8$, as shown in Fig.1(b). Moreover, as τ increases, both the equivalent stiffness coefficient and the negative damping coefficient in absolute of the subsystems increase. At some τ value where the energy increased by unstable subsystems equals to the energy reduced by switching, a periodic solution is created. Figure 1(c) show that two periodic solutions are identified.



(a) $\tau = 0.1$ (b) $\tau = 0.5$ and $\tau = 0.8$ (c) Periodic solutions

Figure 1: (a) System is stable at $\tau = 0.1$ (b) System state at $\tau = 0.5$ or 0.8 (c) Two periodic solutions

References

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