periodic solution of Mathieu system induced by fuzzy uncertainty

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Abstract. The Mathieu system with fuzzy uncertainty is described by fuzzy differential equation in the form of differential inclusions. After transforming the fuzzy differential equation into the governing equation of the membership functions, the membership distribution of the fuzzy solution is solved numerically. It is found that fuzzy uncertainty is able to induce the multiple periodic response around the deterministic periodic solution.

Introduction

The Mathieu equation was introduced by Mathieu to research the oscillation of elliptic membrane. It is rich in the dynamics and can characterize many engineering problems. For the practical engineering problem, the system is inevitably subjected to uncertainties. For the uncertainty with incomplete or vague information, it is often modeled as fuzzy uncertainty. Thus, it has practical significance to study the dynamics of the Mathieu system with fuzzy uncertainty.

Consider the Mathieu system with fuzzy uncertainty as below

$$\begin{cases} \dot{x}_1(t) & = x_2 \\ \dot{x}_2(t) & \in -\left\{25x_1^3 + 0.173x_2 + [2.62 - 0.456[W]^\alpha(1-\cos 2t)]\, x_1\right\} + 0.92[W]^\alpha(1-\cos 2t) \end{cases} . \tag{1}$$

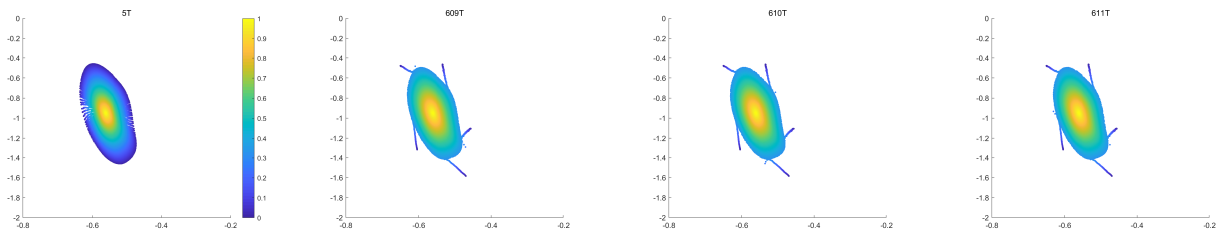
where W is a triangular fuzzy number with the membership functions as

$$\mu_W(w) = \begin{cases} 1 - 20|w - 3.9|, & 3.85 < w < 3.95 \\ 0, & \text{otherwise} \end{cases} ,$$

and $\boldsymbol{x}(0) \in [\boldsymbol{X}_0]^\alpha$. $\boldsymbol{X}_0$ is a fuzzy set with a cone membership distribution centered on the point (a, b) as below

$$\mu_{\boldsymbol{X}_0}(\boldsymbol{x}(0)) = \begin{cases} 1 - \frac{\sqrt{(x_1(0)-a)^2+(x_2(0)-b)^2}}{0.02}, & \sqrt{(x_1(0)-a)^2 + (x_2(0)-b)^2} < 0.001 \\ 0, & \text{otherwise} \end{cases} . \tag{2}$$

Let $T = \pi$, the corresponding deterministic Mathieu system has a period-1 attractor near $(-0.5615, -0.9493)$ and a period-3 attractor. Take $(a, b) = (-0.5615, -0.9493)$ to make the initial condition almost sit at the deterministic period-1 attractor. the method proposed in [1] is used to transfer Eq.(1) into the governing equation of the membership functions. The numerical response solution is shown in Fig. 1. From Fig. 1(b-d), it can be seen that five arms run out from the fuzzy solution and hold steady after a long time evolution. In fact, each one of the five arms keeps rotating around the deterministic period-1 attractor and overlaps all five arms every 5π, that is the arms of the response solution have the period of 5π although the other parts have the same period of π as the deterministic period-1 attractor.



(a) (b) (c) (d)

Figure 1: The fuzzy responses of the Mathieu system with fuzzy uncertainty at different moments, where different colors from blue to yellow represent different values of $\mu_{\boldsymbol{X}(t)}(\boldsymbol{x})$ from 0 to 1. (a)$t = 5\pi$ (b)$t = 609\pi$ (c)$t = 610\pi$ (d)$t = 611\pi$

Results and discussion

The numerical solution shows that fuzzy uncertainty in the Mathieu system can lead some parts of the response solution to have different period from the deterministic attractor, although other parts hold the period of the deterministic attractor.

References

[1] Liu Xiao-Ming, Jiang Jun, Hong Ling (2020) A numerical method to solve a fuzzy differential equation via differential inclusions. *Fuzzy Sets and Systems* **https://doi.org/10.1016/j.fss.2020.04.023**.