# Stochastic bifurcations in three-dimensional piecewise smooth systems

**Mahashweta Patra** and Sayan Gupta
*mahashwetapatraphys@gmail.com*
*Department of Applied Mechanics*
*Indian Institute Technology Madras, Chennai 600 036, India*

**Abstract**. In this work, stochastic bifurcations are studied where various kind of noise is present in the three-dimensional piecewise smooth system. As the switching border moves randomly the piecewise smooth continuous map becomes discontinuous. Thus systems with randomly moving border and discontinuous systems show similar bifurcation phenomena. This paper further shows that when the noise term is present in the functional form of the deterministic map, the noise term affects only the subsequent iterations of the map but does not affect the position of the border. As a consequence, the effect of noise can be seen throughout the bifurcation diagram. The effects of a stochastically moving border are seen only near the border collision bifurcation (BCB). In the case of randomly moving border, the bifurcation type may change but in case of additive noise situation, the bifurcation type remains the same.

## Introduction

Stochastic bifurcations have been thoroughly studied in smooth systems, both in continuous systems and discrete time dynamical systems. However, there are a few research work that involve both piecewise-smooth systems and noise [1, 2]. However, there may arise situations where the border is randomly moving due to the presence of some special kind of noise in the system. Such variation of the border may be caused due to mechanical reasons (loosely fitted panels or dividers), due to variations in the environment (effects of change of pressure or temperature), due to some defined rule of measurements. We have done a comparative study where various types of noise is present in the 3D piecewise smooth system. Effects of multiplicative parametric noise on the system dynamics need to be studied. We keep it as a future work.
A 3D piecewise smooth continuous map in the neighborhood of the border can be approximated [3–5] as

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \begin{cases} \begin{pmatrix} \tau_l & 1 & 0 \\ -\sigma_l & 0 & 1 \\ \delta_l & 0 & 0 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, & x_n \leq 0, \\ \begin{pmatrix} \tau_r & 1 & 0 \\ -\sigma_r & 0 & 1 \\ \delta_r & 0 & 0 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, & x_n \geq 0. \end{cases} \tag{1}$$

## Results

Fig. 1 illustrates period doubling bifurcation with and without the presence of noise. Fig. 1(a) shows a period doubling bifurcation for the deterministic system. Fig. 1(b) illustrates the bifurcation diagram when the border is no longer remain fixed, it moves randomly. Fig. 1(c) shows the situation where the noise is present in the functional form of the system. In this case it is present as an additive term in the x-variable.

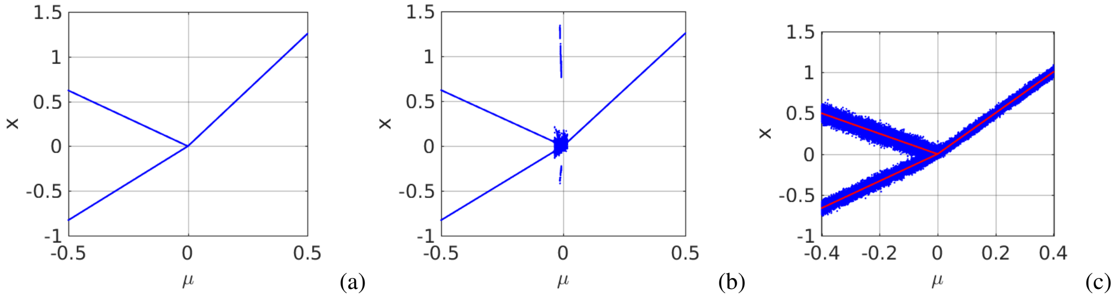


Figure 1: Bifurcation diagram. (a) Piecewise smooth continuous map, (b)when the border is moving stochastically, (c) when the noise is present in the functional form as an additive term. Parameter settings: $\tau_l = -1.3$, $\sigma_l = -1.15$, $\delta_l = -0.36$, $\tau_r = 0.36$, $\sigma_r = 0.4$, $\delta_r = 0.642$.

## References

[1] D. Mandal and S. Banerjee, Physics Letters A **381**, 2161 (2017).
[2] P. Glendinning, SIAM Journal on Applied Dynamical Systems **13**, 181 (2014).
[3] M. Patra and S. Banerjee, Chaos: An Interdisciplinary Journal of Nonlinear Science **28**, 123101 (2018).
[4] M. Patra, International Journal of Bifurcation and Chaos **28**, 1830032 (2018).
[5] I. Roy, M. Patra, and S. Banerjee, Chaos, Solitons and Fractals **133**, 109655 (2020).