Modelling and analysis of vibrations on an aerial cable car system with moving mass

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Abstract. The main purpose of this work is to study the non-linear dynamics of an aerial cable-car. This type of transport is present in thousands of installations, Switzerland has alone more than 130 in operation. The mechanical system presented consists in to two cables, traction cable and supporting cable. The car is modelled as a concentrated mass pulled by a traction cable. The mathematical model is described as a mass-spring system attached to a time varying length cable. The effects of the pulling speed shall generate nonlinear features due to the coupling between the cable and the car and due to external excitation.

Introduction

The aerial cableways, Figure 1a, have been in use for many different applications *e.g.* transport of people and goods, but there are not many published works about them. A proposed mechanical system is illustrated in Figure 1b, where the overhung cable has a constant value of tension by T and is fixed on both ends. Later, it is simplified as a spring k attached to the concentrated mass M representing the car. The car is pulled by the traction cable. However, the length of the traction cable varies in time with constant velocity v. In order to

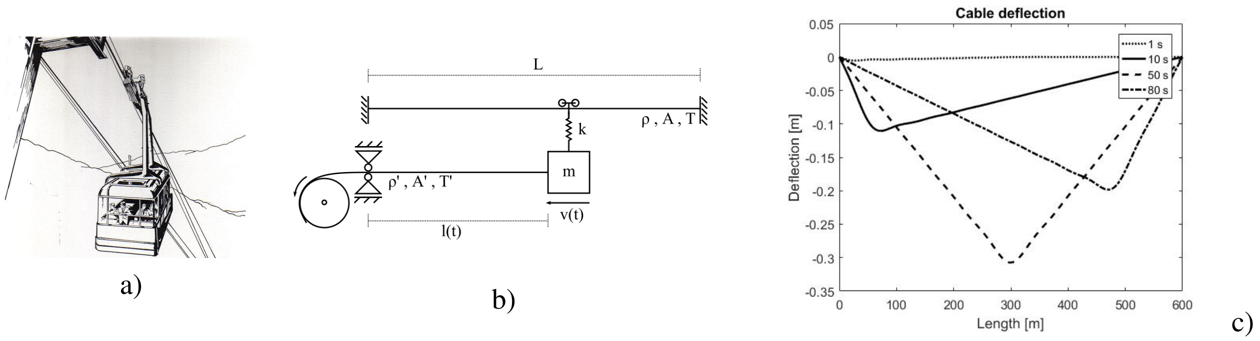

Figure 1: a) A cable car. b) The mechanical system. c) Deflection of the cable due to a moving load, with linear density A = 39.21 kg/m, length L=600 m, load F=25 KN, velocity v=6 m/s, and mean tension T=23.87 MPa.

find the equivalent stiffness of the rail cable, one needs to use the solution found in Hagedorn (2007) of the deflection of a cable with a constant velocity moving load. Its analytical solution is illustrated in Figure 1b for different time steps using the mentioned parameters. For the traction cable, using the Hamilton principle with the adequate definition of the boundaries conditions and considering the variation of the length gives the equations of motions.

$$\rho\left(\ddot{y}+2v\dot{y}+v^2y''\right)-Ty''=0,\ \ 0\le x<l\left(t\right), \tag{1}$$

$$M\left(\ddot{y}+2v\dot{y}+v^2y''\right)\big|_{x=l(t)}+T\,y'\big|_{x=l(t)}+Mg+K\,y\big|_{x=l(t)}=0, \tag{2}$$

where the $\dot{y}$ is the derivation in time and y' is the spatial derivative. A slow time variation $\tau=\varepsilon t$ is introduced, in which $\varepsilon=v/\omega_0 L_0$ and where ω_0 is the natural frequency of the cable without mass and spring and L_0 is the maximum extent of the cable. Knowing that $l(t)=L_0+vt$, so, $\dot{l}=v=\frac{\partial\tau}{\partial t}\frac{\partial l}{\partial\tau}=\varepsilon l$. Therefore, we seek solutions of the following form: $y\left(x,t\right)=\sum_{n=1}^{N}\Psi_n\left(x,\tau\right)q_n\left(t\right)$, where $\Psi(x,\tau)=\sin\left(\beta_n\left(\tau\right)x\right)$. The term $\beta_n(\tau)$ is a slow variation in time of the natural frequency of the cable.

Results and Discussions

This paper's main contribution is to express the cable-car's the dynamical model of a car being pulled by a traction cable and simultaneously suspended by another cable working as rails and to show the representation of the arising nonlinearities that are present and observable on a traction cable from a real case, that can lead to hazardous scenarios.

References

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