Fractional order SIR epidemic model for COVID-19

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Abstract. In this paper, an attempt has been made to study and investigate a non-linear, non-integer SIR epidemic model by incorporating Beddington-De Angelis incidence rate for COVID-19 and Holling type II saturated cure rate with memory. The caputo form of non-integer order derivative is considered. We observe that the model is well posed i.e., the solution with a positive initial value is reviewed for non-negativity and boundary. Basic reproduction number R_0 is determined by next generation matrix method. Routh Hurwitz criteria is used to determine the presence and stability of equilibrium points and then stability analyses are conducted. We observe that the disease-free equilibrium Q^d is stable for\ R_0<1 i.e. there will be no infection in the population and the system tends towards the disease free equilibrium Q^d and for R_0>1, it becomes unstable and the system will tends towards endemic equilibrium Q^e. Lastly numerical simulations to assess the efficiency of mathematical work are eventually performed.

Introduction

In 1927, Kermack and McKendrick developed the first compartmental model for the study the disease dynamics, many researchers then presented numerous mathematical models such as SIR, SIRS, SEIR, SEIRS etc. Since, the 2019 novel coronavirus is discovered, numerous research in modeling COVID-19 transmission have used ordinary differential equations to study its dynamics [1,2] though there are reasons that the non-integer models give better fit to the actual data, numerous models have been developed to study the dynamics of COVID-19 using fractional order derivatives [3,4]. Motivated by the above-mentioned literature review, we formulated a fractional order SIR model for COVID-19 pandemic with nonlinear incidence rate known as Beddington-DeAngelis type incidence rate and Holling type II saturated treatment rate. The fractional order derivative is taken in Caputo-sense. Whole population N(t) is categorized in three classes namely susceptible S(t), infectives I(t) and recovered R(t). We assume that recovered cannot become infected again and, they cannot infect the susceptible population again. Under these assumptions we proposed a Caputo- fractional order SIR model. The disease -free equilibrium and endemic equilibrium have been analyzed for the developed model and reproduction number has been calculated.

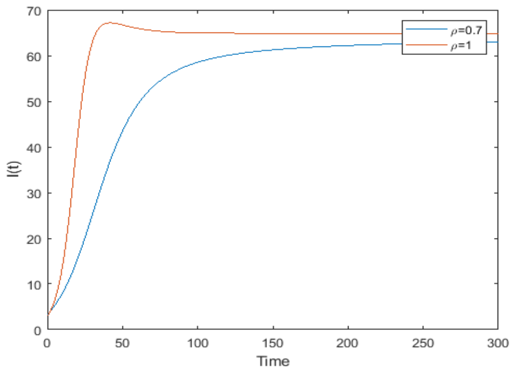

Figure 1: Result of changing fractional order ρ on infectives.

Results and Discussion

Simulation has been carried out with the help of MATLAB. It is noted that varying the values of non-integer order derivatives \rho does not impact the equilibrium stability of the system but the time to achieve steady conditions or endemic equilibrium decreases, i.e., better convergence can be accomplished by reducing the order of fractional derivatives or greater the values of ρ slower is the convergence thus, the utility of the non-integer order derivative rather than the integer order model has been investigated. With this fractional order model and taking the authentic data of COVID-19, the dynamics of COVID-19 can be predicted.

References

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