Properties of the motion of several interacting bodies under periodic excitation

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Abstract. The report considers a system of several interacting bodies, that move along a line in a resistive media under periodic excitation. Periodic by velocity regimes of motion are studied. Existence, uniqueness and exponential stability of such periodic regimes of motion are proven. For a system of two bodies, the results are extended to the cases of dry friction, collisions and planar motions.

Introduction.

Rectilinear movement of n interacting bodies is considered. The motion of the system is induced by a periodic excitation. That is, the distances between the main body with mass m and the attached bodies with masses m_i change periodically, $l_i(t+T) = l_i(t)$, $i = 1, \dots, n-1$. Body m and bodies m_i are subjected to the environment resistances r and r_i, which are generally different (see Fig. 1). If one of the resistances is zero, then the system can be regarded as capsule system with several attached bodies.

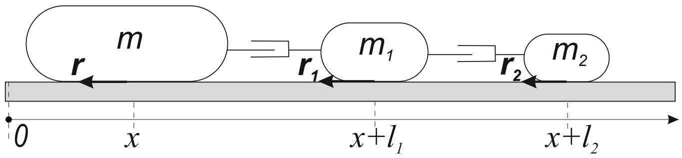

Figure 1: System of interacting bodies, $n = 3$.

The motion of the system is governed by the equation

$$m\ddot{x} + \sum_{i=1}^{n-1} m_i(\ddot{x} + \ddot{l}_i) = r(\dot{x}) + \sum_{i=1}^{n-1} r_i(\dot{x} + \dot{l}_i)$$

We wonder if there exist initial conditions $\dot{x}(0)$, under which the velocity of the main body is periodic by time, $\dot{x}(t+T) = \dot{x}(t)$. This is equivalent to the periodicity of the velocities of all bodies $\dot{x}_i(t+T) = \dot{x}_i(t)$. If the velocity is periodic, the system travels the same distance for each time period. We are also interested if such motion is unique and if all solutions $\dot{x}(t)$ tend to the periodic one, i.e. this motion is stable.

The described above and similar systems controled by interactions between the bodies were considered in a large number of papers, especially for $n = 2$, see [1]. Meanwhile, stability of periodic by velocity motions for such systems was studied only for some specific cases [2, 3]. The proof technique introduced in the current research allows to solve this question for a system consisting of several interacting bodies and for rather wide variety of media resistance laws.

Results and discussion

We assume that media resistances r, r_i are defined by monotonically decreasing functions of the corresponding velocities, $|r|$ goes to infinity when $\dot{x} \to \pm\infty$, and relative accelerations $\ddot{l}_i(t)$ are continuous functions. The following proposition is proved.

Proposition. The motion with periodic velocities exists, is unique and is exponentially stable.

In case of two bodies with second body not subjected to media resistance, $n = 2$, $r_1 = 0$ (this is the case of a capsule system), additional results are obtained. Firstly, this proposition remains true for discontinuous relative accelerations, i.e. when collisions are allowed. Secondly, the proposition takes place in the case when media resistance r for the main body is defined by dry friction and the relative acceleration $\ddot{l}_1(t)$ is sufficiently big providing that each body never stays at rest.

In the case of linear resistance, two-dimensional motion of two interacting bodies is considered. We prove, that the motion of the system tends to a rectilinear motion and the proposition can be applied to this rectilinear motion.

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References

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