Perturbation approach to a cellular metamaterial with embedded vibration absorbers

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Abstract. Metamaterials with an engineered cellular arrangement of functionalized mechanical devices exhibiting amplitude-dependent dispersion properties are attracting increasing interest in the research field of periodic microstructures. The paper investigates the wavefrequencies and waveforms of a periodic waveguide with local vibration absorbers, characterized by weak geometric nonlinearities and nonlinear visco-elastic damping. The approach uses tools borrowed from Hamiltonian Perturbation Theory as well as techniques classically employed in the context of nearly-integrable Hamiltonian systems in order to asymptotically describe the solutions. The zero-dissipation limit case is presented for comparison with the literature. The case of weak dissipation is also discussed.

Introduction

The band structure of microstructured periodic waveguides is of wide interest in nonlinear dynamics, especially regarding the amplitude-dependent pass and stop bands of oscillator chains and other periodic structures [1]. Extensive efforts are currently targeted at designing mechanical cellular metamaterials characterized by large-amplitude band gaps with tunable low center-frequency. To this end, intracellular mechanisms of local resonance can be realized by highly-flexible, massive and damped resonators working as propagation inhibitors and energy absorbers. This research stream may promote understanding and exploitation of important dynamic phenomena determined by geometric and constitutive nonlinearities.

Results and discussion

A minimal locally resonant system can be realized by an infinite one-dimensional dissipative chain of rigid rings, embedding geometrically nonlinear oscillators [2]. In order to improve the passive control performance [3], the mechanical model can be enriched by adding nonlinear visco-elastic dissipation which allows better tunability of the amplitude-dependent damping (Figure 1). The dynamic problem of wave propagation is governed by a coupled system of non-linear ODEs. From the methodological viewpoint, the perturbative analysis of the governing equations is carried out in the neighbourhood of a hyperbolic equilibrium for the system after a suitable rescaling. The employed technique relies on the possibility to interpret a given system of ODEs as a Hamiltonian system in a suitably extended phase space. Accordingly, the associated Hamiltonian function can be directly treated by using well established tools of Perturbation Theory for nearly-integrable Hamiltonian systems. Canonical transformations of variables generated through Lie-series operators are employed, with an approach already profitably used to deal with the theory of normal forms for general systems of ODEs with time-dependent nonlinearities [4]. Under mild non-resonance hypotheses on the eigenvalues of the linearised system, the (first-order) correction to the linear frequencies arise naturally from the normal form construction. A comparison with literature results corresponding to the zero-dissipation limit is discussed, as well as a case of weak dissipation by examining collateral perturbative scenarios. Within this context, special attention is devoted to the issue of constructing invariant manifolds of motion under the prevailing assumptions.

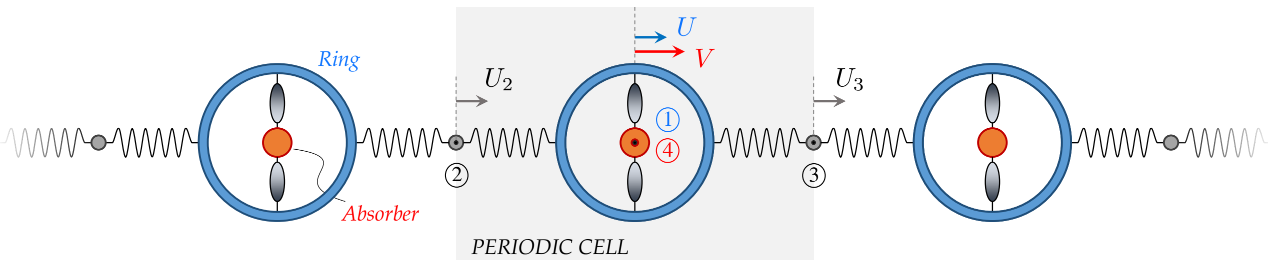

Figure 1: Cellular metamaterial with embedded locally-resonant vibration absorbers.

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