Escape of excited conservative 2DOF system from potential well

Oleg Gendelman*, Tal Ezra* and Alexander Fidlin**

*Faculty of Mechanical Engineering, Technion – Israel Institute of Technology, Haifa, Israel

**Institute of Engineering Mechanics, Karlsruhe Institute of Technology, Karlsruhe, Germany

Abstract. We address the problem of the escape of a dynamical system with two degrees of freedom from a one-dimensional potential well. Dynamics of the escape process is governed by energy exchange between internal degrees of freedom of the system and the potential well. The main object of the study is the relationship between critical initial energy required for the escape, and internal coupling in the system. Asymptotic regimes of small and large coupling are tractable analytically. Quite surprisingly, these two limit cases reliably cover almost the whole range of possible coupling values. In addition, it is shown that the system between the zones of validity of these two limit asymptotic approximations exhibits chaotic characteristics, and the energy threshold is close to a possible minimum due to presumably ergodic dynamics of the system.

Introduction

Escape from a potential well is a classic problem, relevant in many branches of physics, chemistry and engineering [1,2]. For the case of one degree of freedom (DOF), there are theoretical criteria for the transition between potential wells, and its agreement with laboratory experiments have been shown [3-5]. When there are two or more DOF, the situation becomes more complicated. We analyze dynamics of a 2DOF system of two particles coupled by a linear spring ("molecule") inside a potential well. The system is excited by applying initial velocity to one of the particles. Three models of potential wells (hyperbolic secant, cubic and biquadratic) are explored. The goal is to establish the relationship between the critical energy required for the escape and the coupling between the particles.

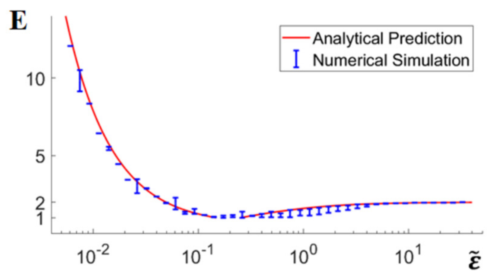

Figure 1: Dependence of critical escape energy on the coupling.

Results and discussion

The treatment is based on consideration of two separate asymptotic regimes. In the case of ***strong*** coupling the characteristic frequency of relative oscillations of the particles becomes relatively high, and thus slow-fast decomposition of the time scales is justified. After averaging, the problem is reduced to trivial analysis of the motion of centre of masses of the "molecule" in a modified effective potential. For the case of ***weak*** coupling, the treatment is based on the consideration of limiting problem: whether the excited particle will be able to pull the second particle out of the well. Both limiting cases yield rather reliable analytic predictions for their appropriate ranges of validity. For the hyperbolic secant potential, the comparison is presented in Figure 1. Quite surprisingly, the two limiting cases cover almost all range of the coupling strength parameter. Small intermediate zone corresponds to chaotic motion of the system in the well. The system, presumably, behaves ergodically and eventually finds a channel for escape in the state space even if the total energy is close to a minimum required for this purpose. All three explored models with different potentials demonstrate very similar qualitative behaviour.

References

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