Investigation of quasi-periodic solutions in nonlinear oscillators featuring internal resonance

Giorgio Gobat*, Cyril Touzé**, Louis Guillot†, Bruno Cochelin† and Attilio Frangi*
**Dipartimento di ingengeria Civile e Ambientale DICA, Politecnico di Milano, Milano, Italy*
***IMSIA, Institut of Mechanical Sciences and Industrial Applications, ENSTA Paris, CNRS, EDF, CEA, Institut Polytechnique de Paris, Palaiseau, France.*
†*Aix Marseille Université, CNRS, Centrale Marseille, LMA UMR7031, Marseille, France*

Abstract. Quasi-periodic solutions arise in assembly of nonlinear oscillators due to a Neimark-Sacker (NS) bifurcation. A detailed analytical and numerical investigation of the appearance of NS bifurcation in coupled oscillators featuring both 1:2 and 1:3 internal resonance is provided. For the 1:2 case, an analytical expression of the locus of NS points is found. For the 1:3 case, a numerical continuation of NS bifurcation points is performed in order to derive the locus of the NS points. The obtained results allows accurate predictions of occurrence of quasiperiodic solutions thanks to the derivation of these boundary curves.

Introduction

This work finds its origin in the numerous reports of appearance of frequency combs in the nonlinear dynamics of Micro Electro Mechanical Systems (MEMS) see *e.g.* [1] and references therein. On the dynamical system point of view, these frequency combs are intrinsically quasi-periodic solutions with two main frequencies and the nonlinear mixing creating the whole comb. Investigating the birth of these solutions is thus related to determining the locus of Neimarck-Sacker (NS) bifurcation points in parameter space. As also known from the theory, internal resonance in nonlinear coupled oscillators favours the appearance of more complex solutions and thus appearance of NS bifurcations. The aim of this study is to derive the locus of NS bifurcation points for two typical two-degrees of freedom nonlinear oscillators displaying 1:2 and 1:3 internal resonance.

Results and Discussion

The starting point of the two studies is the normal form of two-dofs oscillator featuring either 1:2 or 1:3 internal resonance. The systems are also considered with damping and forcing and the external forcing is harmonic, in the vicinity of the low-frequency mode. In each case, the appearance of quasiperiodic (QP) solutions has already been reported in the past, but, to the best of our knowledge, the locus of the NS bifurcation points has not been computed explicitely before.

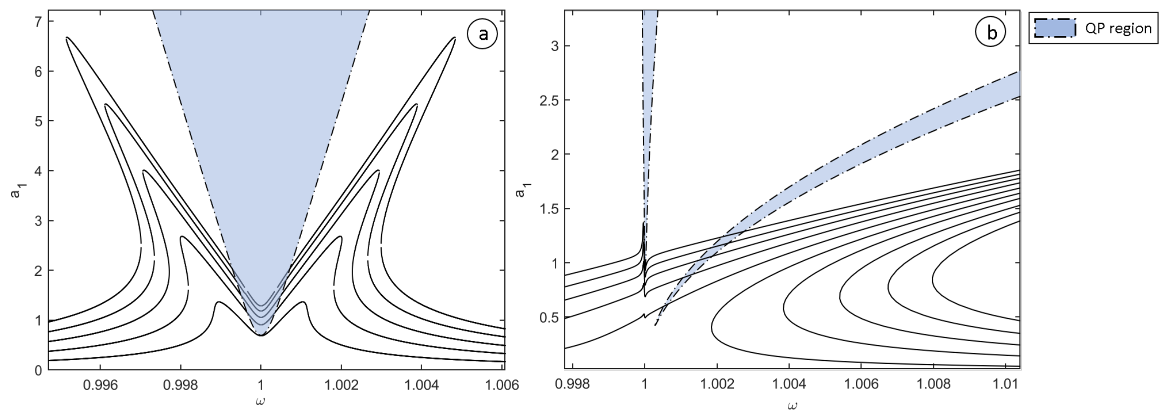

Figure 1: Frequency Response Function (FRF, black lines) of 2 dof oscillators featuring (a) 1:2 and (b) 1:3 internal resonance along with their NS bifurcation region (dash dot boundary and shaded light blue). Each FRF corresponds to a different forcing level. When the amplitude of motion becomes large enough, the NS boundary is crossed by the FRF and the dynamics becomes quasi-periodic.

Fig. 1 shows the results obtained for the 1:2 case and 1:3 case. For the 1:2 internal resonance, an analytical expression has been derived, expressing the boundary curve of QP solutions as functions of all the parameters of the problem. For the 1:3 internal resonance, the analytical derivation becomes too difficult arising in lengthy polynomial expressions with high orders. Consequently, a numerical approach has been developed to derive the boundary curve. It is based on the numerical continuation of NS bifurcation points, and has been implemented in the software MANLAB [2, 3]. Fig. 1(b) shows the two regions appearing in a system with 1:3 resonance.

References

[1] D. A. Czaplewski, C. Chen, D. Lopez, O. Shoshani, A. M. Eriksson, S. Strachan and S. W. Shaw (2018). Bifurcation Generated Mechanical Frequency Comb. *Physical Review Letters*, **121**, 244302.

[2] Guillot, L., Cochelin, B., & Vergez, C. (2019). A Taylor series-based continuation method for solutions of dynamical systems. *Nonlinear Dynamics*, **98**(4), 2827-2845.

[3] Guillot, L., Lazarus, A., Thomas, O., Vergez, C., & Cochelin, B. (2020). A purely frequency based Floquet-Hill formulation for the efficient stability computation of periodic solutions of ordinary differential systems. *Journal of Computational Physics*, 109477.