Analytical approximation of forced oscillations of nonlinear Helmholtz Resonator by Homotopy Analysis Method

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Abstract. This study investigates nonlinear amplitude-frequency response of an acoustic nonlinear Helmholtz Resonator (HR) by taking into account nonlinear restoring and damping forces. For this kind of nonlinear resonator, it was shown in [1] that two regimes can be obtained: softening behavior first (for low levels) and then hardening behavior (for high levels). In function of the order of magnitude of displacement, it is possible to introduce a small parameter and these two regimes can be modelized separately by using classical multiple scales method developped by Nayfeh [2]. In the present paper, we derive analytical expression applying the Homotopy Analysis Method (HAM) introduced by Liao [3]. Without any need to change assumptions on order of magnitudes of system parameters, this method allows us to obtain the nonlinear frequency responses for small, intermediate as well as large amplitude vibrations.

Introduction

Nonlinear acoustic HR can provide wide possibilities of practical applications, for example targeted nonlinear energy transfer in acoustics [4]. Nonlinear character of restoring characteristic and damping part can lead to substantial improvement of this element. So it becomes important to be able to describe and predict the nonlinear amplitude-frequency response analytically to be able to optimize and design the HR for practical applications. However, this kind of nonlinear resonator involve quadratic and cubic terms in the nonlinear restoring force which lead to different regimes (softening or hardening behaviour) in function of amplitude of excitation. Those behaviors can be well predicted as shown in [1] but the methods usually require the use of a small parameter. So the problem is often solved as a wealy nonlinear system involving only small finite amplitudes or as strongly nonlinear system involving strong amplitudes. That is why we propose in the present study to use HAM to obtain the whole frequency response curves for various values of parameters. We also have to take into account the mean of motion which can not be disregarded because the damped system involves quadratic and cubic nonlinearities. This last point can be performed following the work developed in [5].

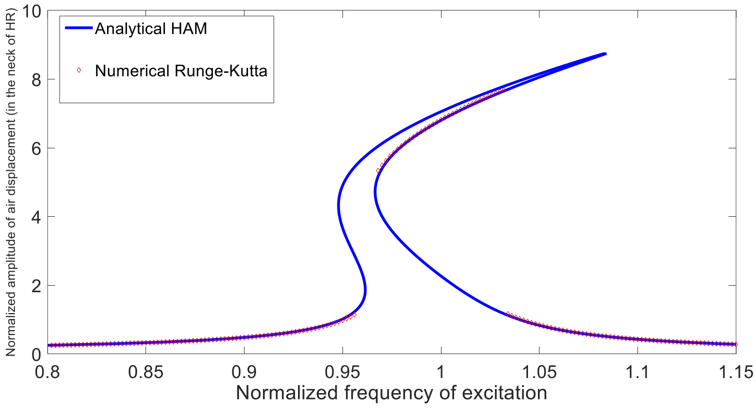

Figure 1: Example of a nonlinear amplitude-frequency response obtained analytically by HAM showing softening and hardening behavior.

Results and discussion

Applying HAM to the forced nonlinear HR allows to obtain nonlinear amplitude-frequency responses for the whole system as shown in Figure 1 exhibiting different regimes (softening or hardening behaviour) with no use of a small parameter. The analytical approximations have been compared with classical Runge-Kutta methods and the accuracy is very good. Various values of the coefficients for nonlinear restoring and damping forces have been used to study the influence of each term.

References

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