Stability of similar nonlinear normal modes under stochastic excitation

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Abstract. Two-DOF nonlinear system allowing nonlinear normal modes (NNMs) with rectilinear trajectories in the system configuration space under white noise excitation is is considered. Influence of the stochastic excitation to the NNMs stability is analyzed using the analytical-numerical test, which is an implication of the known Lyapunov stability criterion.

Introduction

Different aspects of the NNMs theory and applications of the theory are presented in numerous publications, in particular, in review [1]. NNMs having rectilinear trajectories in configuration space (so-called similar NNMs) were first described by Rosenberg [2]. Numerous publications are dedicated to investigation of dynamical systems under stochastic excitation, in particular, the books [3,4]. Behavior of nonlinear systems under random inputs is studied using the NNMs concept in few recent publications, in particular, in [6]. Here some numerical-analytical test [6] which is a consequence of the Lyapunov criterion of stability is used to analyze the stability of the similar NNMs in two-DOF nonlinear system under white noise excitation.

Stability of the similar nonlinear normal modes

The homogeneous two-DOF nonlinear system under stochastic excitation is considered. The system without such excitation allows two similar NNMs, namely, the in-phase and out-of-phase NNMs. We consider the external excitations which save these NNMs but can change their stability. We use the following stability test [6], which is a consequence of the Lyapunov criterion of stability: Instability of the solution $y = 0$ is fixed if

$$|y(t)| \geq \rho\ |y(0)| \qquad (0 \leq t \leq T). \tag{1}$$

In fact, in the instability region the variations leave the solution ε-neighborhood under increase of t for any choice of the parameter ρ. We can choose, for example, $\rho = 10$. In contrast to the Lyapunov definition a time T is limited in the test (1). Note that concrete calculations are realized at points of some chosen mesh in the parameter space. Calculations are conducted as long as boundaries of stability/instability regions in the space are variable. This is a criterion for the choice of the time T [6]. Taking into account a specific character of the stability problem under stochastic excitation, we introduce some modification to the test (1). Namely, in the last case some variations can leave the ε-neighborhood of the solution and then return back to one. Thus, we allow that some small part of variations is out of the neighborhood during each fixed interval of time.

Results and Discussion

Figs. 1 and 2 demonstrate boundaries of the stability/instability regions of the out-of-phase NNM for some system parameters. Here the parameter ε characterizes a smallness of the excitation; γ characterizes a connection between partial oscillators of the system. Direct numerical simulation confirms these results. Numerical simulation shows also that the determined chaotic excitation does not affect to the NNMs stability. Thus, the test (1) is effective in problem of stability of NNM under stochastic excitation.

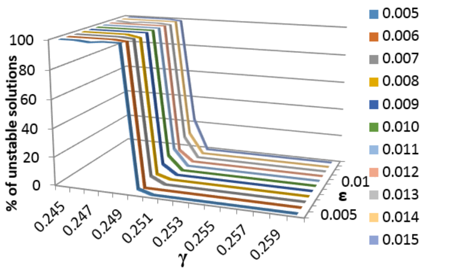

Figure 1:. Change of the out-of phase NNM stability parameters γ and ε

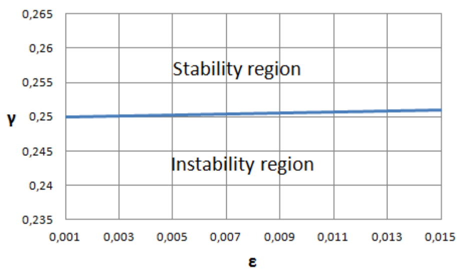

Figure 2: Boundaries of the stability/ versus to instability regions in place (γ,ε)

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