Analysis of the steady states stability using the Ince algebraization

Yuri V. Mikhlin[*], Natalia S. Goloskubova[*]

[*] *Department of Applied Mathematics, National Technical University "KhPI", Kharkiv, Ukraine*

Abstract. A stability of the nonlinear normal modes (NNMs) and traveling waves is studied by the Ince algebraization (IA), when a new independent variable associated with the solution under consideration is used as independent one. An advantage of the IA is that in the stability problem we do not need in use of the unperturbed solution time-presentation.

Introduction

Different approaches can be used to solve a problem of the modes or stationary waves stability [1,2]. The stability can be effectively analyzed by the so-called *Algebraization by Ince* (IA) [3]. This approach is performed by choosing a new independent variable associated with the solution under consideration. An advantage of the IA is that we do not need in use in the stability analysis of specific form of the solution. The IA was successfully used earlier in a problem of the NNMs stability [4,5]. A stability of NNMs in the system of connected oscillators on grounding elastic support under conditions of the so-called sonic vacuum [6] is considered. Dynamics of the system is describes as

$$\begin{cases} \mu \dfrac{d^2 v_1}{dt^2} + {v_1}^3 + \dfrac{\mu}{6}\left[{v_1}^2 + (v_2 - v_1)^2 + {v_2}^2\right](2v_1 - v_2) = 0 \\ \mu \dfrac{d^2 v_2}{dt^2} + {v_2}^3 + \dfrac{\mu}{6}\left[{v_1}^2 + (v_2 - v_1)^2 + {v_2}^2\right](2v_2 - v_1) = 0 \end{cases} \tag{1}$$

Using the IA and introducing a new independent variable z, describing a motion along the NNM, instead of t, one obtains the equation in variations in the form of the linear equation with singularities. In this case solutions corresponding to boundaries of the stability/instability regions in the system parameter space, are determined by the following expansions [3-5], where r is one of two indices of the singular point:

$$u = z^r(a_0 + a_1 z + \cdots). \tag{2}$$

The same approach is used also in problem of the stability of stationary traveling waves, $u = \Phi(\varphi)$, $\varphi = kx - \omega t$, in the Klein-Gordon equation with cubic nonlinearity:

$$\frac{\partial^2 u}{\partial t^2} - {c_0}^2 \frac{\partial^2 u}{\partial x^2} + {\omega_0}^2 u = -qu^3 \tag{3}$$

Results and discussion

Boundaries of the stability/instability regions are shown in Fig.1 for the considered stationary states. It shows a stability of NNMs in place *(h ,μ)* (Figs. 1a, 1b) and of the traveling wave in place (*B*,*h*) (Fig.1c), where *h* characterizes the state energy, the parameter *B* presents some system and the wave characteristics. Regions of instability are situated above the boundaries in Figs.1a, 1b, and between the boundaries in Fig.1c.

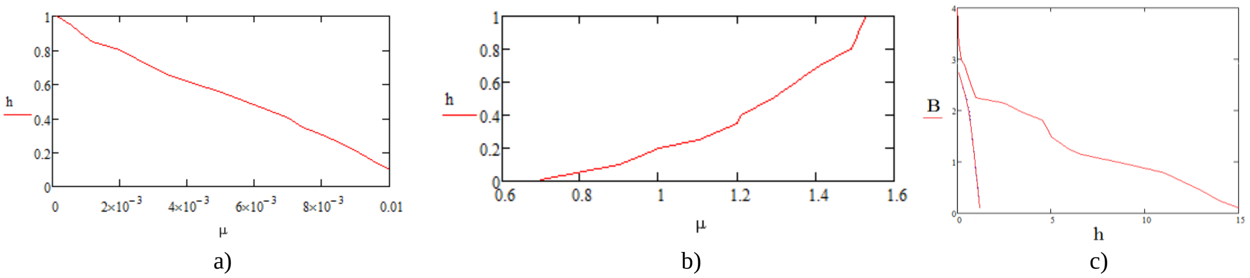

Figure 1. Boundaries of regions of stability/ instability on place *(h ,μ)* for the first NNM of the system (1) (Fig. 1a), for the second one (Fig. 1b), and for the traveling wave (Fig. 1c) .

Note that direct numerical simulation for solutions chosen from the stability/instability regions confirms obtained results. It demonstrates an effectivity of the Ince algebraization for the stability problems.

References

[1] LaSalle J.P. (1976) The Stability of Dynamical Systems. SIAM, Philadelphia.
[2] Glendinning P. (1994) Stability, Instability and Chaos. Cambridge Univ. Press, NY.
[3] Ince E.L. (1926) Ordinary differential equations. Longmans Green, London.
[4] Mikhlin Yu.V., Avramov K.V. (2010) Nonlinear normal modes for vibrating mechanical systems. Review of theoretical developments. *Appl. Mech. Rev*. **63** (6): 060802.
[5] Mikhlin Yu.V., Zhupiev A.L. (1997) An application of the Ince algebraization to the stability of non-linear normal vibration modes. *Int. J. of Non-Linear Mechanics* **32** (2): 393-409.
[6] Koroleva (Kikot) I., Manevitch L., Vakakis A.F. (2015) Non-stationary resonance dynamics of a nonlinear sonic vacuum with grounding supports. *J. of Sound and Vibration* **357:** 349-364.