A generalized solution scheme using an implicit time integrator for piecewise linear and nonlinear systems

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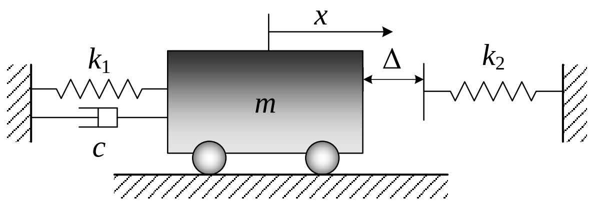
Abstract. A generalized solution scheme using an implicit time integrator for piecewise linear and nonlinear systems is developed. The piecewise linear characteristic has been discussed in previous studies, which transform the problem into linear complementarity problems (LCPs), and solve the LCPs via the Lemke algorithm per step. In the proposed scheme, the projection function is used to describe the discontinuity in the dynamics equations, and the discrete nonlinear equation per step of the implicit integrator is solved by the semi-smooth Newton iteration. Compared with the LCP-based scheme, the new scheme is more general, since it allows other nonlinearities in the dynamics equations. Numerical results of illustrative examples indicate that the new scheme, where the generalized-α method is used as the implicit integrator, has obvious efficiency advantage on the LCP-based scheme for piecewise linear systems, and can also present satisfactory numerical solutions for piecewise nonlinear systems.

Introduction

Piecewise linear and nonlinear systems, such as gap-activated springs in vibrating machines, gear backlash, structures with damage or clearance, and drag torques, can be commonly found in civil engineering, aerospace, mechanical engineering, and infrastructures. They exhibit complex and diverse dynamical behaviours due to the segmentation feature, but this kind of nonlinearity also brings difficulties to the accurate simulation of dynamic response. Among the available methods, implicit time integrators can be considered as better choices because of their high accuracy and robust stability. In their procedures, the current states are updated from the previous ones according to the recurrence scheme, and the discrete nonlinear equations need to be solved by a nonlinear solver per step. Based on the Bozzak-Newmark method [1], Yu [2] developed the basic idea of the LCP-based scheme, but this scheme is applicable only when the piecewise linear characteristic is the only nonlinearity in the dynamics equation. As a further step, this work aims at providing a generalized solution scheme using implicit time integrator for piecewise linear and nonlinear systems.



LCP-based scheme
$k(x) = k_1 x + k_2 y$
$g = y - x + \Delta, y \geq 0, g \geq 0, y \bullet g = 0$
By using the projection function
$k(x) = k_1 x + k_2 \text{proj}_{\mathbb{R}_0^+}(x - \Delta)$

Figure 1: Piecewise linear SDOF system and the formulations for its stiffness term $k(x)$

Formulation

For illustrative purpose, the piecewise linear single-degree-of-freedom (SDOF) system, as shown in Fig. 1, is considered. Its dynamics equation can be written as

$$m\ddot{x} + c\dot{x} + k(x) = 0,\ k(x) = \begin{cases} k_1 x, & x \leq \Delta, \\ k_1 x + k_2(x - \Delta), & x > \Delta. \end{cases} \tag{1}$$

By using the projection function, this equation can be equivalently rewritten as

$$m\ddot{x} + c\dot{x} + k_1 x + k_2 \text{proj}_{\mathbb{R}_0^+}(x - \Delta) = 0,\ \text{proj}_{\mathbb{R}_0^+}(x - \Delta) = \begin{cases} 0, & x \leq \Delta, \\ x - \Delta, & x > \Delta. \end{cases} \tag{2}$$

The projection function presents a more elegant way to express the discontinuity. More importantly, the discrete Equation (2) at each step can be solved by the semi-smooth Newton iteration. Thus, the dynamics equation can contain other nonlinearities, which are not allowed in the LCP-based scheme with an implicit integrator. In a straightforward manner, the formulation can be extended to multi-degree-of-freedom (MDOF) systems with multiple piecewise linear or nonlinear characteristics. Owing to the fast convergence rate of the Newton iteration, the new scheme is more efficient than the LCP-based scheme when applied to systems with a large number of piecewise linear features. Besides, it can also predict accurate results for piecewise nonlinear problems.

References

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