Influence of Temperature on Bifurcation Analysis of Suspended cables Close to Two-to-one Internal Resonance

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Abstract. The influence of temperature on internal resonances of suspended cables with quadratic and cubic geometric nonlinearities are investigated, limiting to the case of two-to-one. A kinematically condensed nonlinear model considering temperature effects is introduced for the in-plane displacements. The method of multiple scales is adopted to present a perturbation analysis of the resonant responses. Numerical results clearly show the influences of temperature changes on nonlinear internal resonances of the system and highlight the significance of the thermal-induced expansion and contraction. Qualitative and quantitative discrepancies induced by temperature variations can be observed for the driven and companion modes. Moreover, results obtained from the method with multi-timescales are in good agreement with those from the direct numerical integrations.

Introduction

Internal resonance is one of the most significant and well-known phenomena in both mechanics and physics. The commensurable eigenfrequencies of suspended cables cause many types of internal resonances associated with complex nonlinear dynamic phenomena [1]. It is known that thermal effects may bring about an unexpected change in mode frequencies. We have investigated the free and forced nonlinear oscillations of suspended cables in thermal environments recently [2]. However, the internal resonance was ignored, and the single-mode discretization was considered. This work investigates the temperature effects on the bifurcations and stability of suspended cables considering two-to-one internal resonances. A horizontal Irvine cable model is considered, and therefore, the bending, torsion, and shear are all ignored, and its strain is induced by stretching and temperature changes. Based on the classical condensed mathematical model, the in-plane PDEs with thermal effects could be obtained by introducing a tension variation factor [3]

$$m\frac{\partial^2 v}{\partial t^2}+c_{\Delta T}\frac{\partial v}{\partial t}-\chi_{\Delta T}^2 H\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}-\frac{EA_{\Delta T}}{L}\left(\frac{1}{\chi_{\Delta T}^2}\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}+\frac{\partial^2 v}{\partial x^2}\right)\int_0^L\left[\frac{1}{\chi_{\Delta T}^2}\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\partial v}{\partial x}+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2\right]\mathrm{d}x=f_v\cos\left(\Omega_v t+\theta_v\right)$$

Introducing the non-dimensional variables and parameters and applying the Galerkin method, one gets the non-dimensional in-plane reduced ODEs. In the perturbation analysis, a small dimensionless bookkeeping parameter is introduced, and an external and an internal detuning parameter are introduced. Following a similar procedure by Srinil and Rega [4], the modulation equations' polar and Cartesian form could be achieved. The equilibrium and dynamic periodic solutions and their stabilities could be determined by using some numerical algorithms.

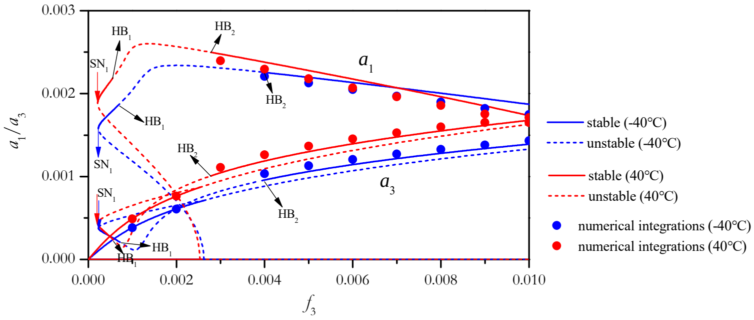

Figure 1: Force-response amplitude curves with thermal effects in the case of two-to-one resonances

Unexpected changes in mode frequencies are induced by thermal effects. Here, the two-to-one internal resonances between the first two symmetric mode frequencies are investigated. The higher-order mode is excited directly, and the lower-mode is internally responded. As shown in Fig. 1, quantitative discrepancies due to thermal effects are presented. The response amplitudes of the driven mode a_3 are reduced significantly in the cooling condition. In contrast, the companion mode response amplitudes a_1 may be increased or decreased, depending on the excitation amplitudes f_3. The bifurcation points SN and HB and resonant ranges are very susceptible to temperature variations. It is observed that the analytical solutions agree well with the numerical ones.

References

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