

# Arresting Motion in Nonlinear Systems Using Two-scale Command Shaping

Alexander Alyukov<sup>\*,\*\*</sup>, Michael J. Leamy<sup>\*</sup>

<sup>\*</sup>George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, USA

<sup>\*\*</sup>Automotive Engineering Department, South Ural State University, Chelyabinsk, Russia

**Abstract.** We present a feedforward technique for arresting motion in nonlinear systems based on two-scale command shaping (TSCS). The advantages of the proposed technique arise from its feedforward nature and ease of implementation in linear and nonlinear systems. Using the TSCS strategy, the control input required to arrest motion is decomposed into two scales: the first arrests dynamics associated with the linear subproblem, while the second eliminates response from the nonlinearities. Using direct numerical integration, the method is assessed using a traditional Duffing system and multi-degree-of-freedom (MDOF) nonlinear systems. Experiments are conducted on a compound pendulum attached to a servo motor, documenting effective arrest of the system in close agreement with theoretical predictions.

## Introduction

Compared to feedback controllers, feedforward methods usually have decreased cost, minimal sensor requirements and faster control response, as they act directly on the input, not on the error signal. One of the promising recently proposed feedforward control method aimed at mitigating undesirable residual vibrations in nonlinear systems is the two-scale command shaping. TSCS is an approach for tailoring a flexible system's applied control input to reduce undesirable residual vibrations using multiple problem scales, input shaping of a linear subproblem, and cancellation of a remaining nonlinear subproblem [1]. Research papers involving traditional input shaping assume the system undergoes rest-to-rest or rest-to-motion. If the system moves from nonzero initial conditions, traditional input shapers cannot be applied due to their open-loop nature. Arrest of a system with nonzero initial conditions can be performed by modified input shapers [2]. However, herein we show that an approach based on TSCS can arrest motion in linear and nonlinear systems using a single rectangular input that outperforms modified input shaping.

## Results and discussions

Using a two-scale expansion [1], the arrest of Duffing systems (a system with cubic non-linearity) is investigated. However, the proposed technique is universal, and can be used for nonlinearities of various kinds. An example of the arrest of coupled Duffing oscillators is considered in which the natural frequencies differ significantly (3.6 times). Figure 1 shows the results obtained for this nonlinear MDOF system using TSCS assessed using direct numerical integration. From this figure, it can be seen that TSCS can effectively arresting the motion of both degrees of freedom,  $x_1(t)$  and  $x_2(t)$ , rapidly and smoothly negating the motion. MDOF systems can be underactuated (the number of control inputs is less than the number of degrees of freedom). In this case, TSCS is also effective, as will be demonstrated in the oral presentation. The presentation will also include discussion of a series of experimental arrests of a pendulum released from a large initial angle (90 degrees), showing close agreement with theoretical results.

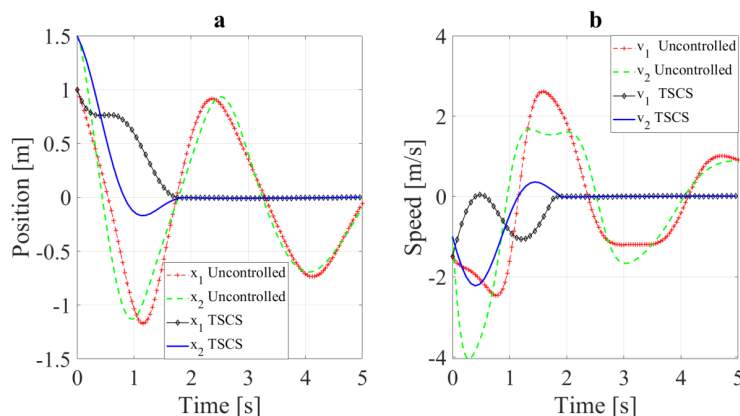


Figure 1: Position (a) and speed (b) of a coupled damped oscillators with cubic nonlinearity subjected to, or absent of, the proposed control technique

## References

- [1] Wilbanks, J.J., Adams, C.J., Leamy, M.J.: Two-scale command shaping for feedforward control of nonlinear systems. *Nonlinear Dynamics* 92:885-903 (2018)
- [2] Newman, D., Hong, S.-W., Vaughan, J.: The design of input shapers which eliminate nonzero initial conditions. *J. Dyn. Syst. Meas. Control* 140, 101005-1-9 (2018)