Nonlinear aspects of one-dimensional supersymmetry

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Abstract. We discuss a number of aspects of nonlinear supersymmetry in the context of predicting the main candidates for beyond the Standard Model physics or understanding the dynamical behavior of strongly interacting quantum systems where perturbation theory is not applicable. We focus on the relation between nonlinear supersymmetry as a natural generalization of linear supersymmetry and generalized statistics, analyzing exotic properties related to quantum anomalies, emergence of hidden symmetries and nonlinear superconformal symmetry. We discuss properties of nonlinear supersymmetry and nonlinear superconformal symmetry and investigate one-dimensional quantized supersymmetric systems with nonlinear features. These properties provide a natural explanation for the anomalous dynamics in quantum systems, leading to various mechanisms in field theory, nonlinear wave physics, cosmology and condensed matter physics.

Introduction

Supersymmetry (SUSY) is an extension of the Standard Model and a conjectured symmetry between Bose and Fermi statistics, providing a natural mechanism for unification of gravity with electromagnetic, strong and weak interactions. Supersymmetric models [1] played a central role in many advances in theoretical physics since the discovery of supersymmetry. Supersymmetric quantum mechanics has been applied for solving many problems in theoretical and mathematical physics, providing a non-perturbative method to explore new iso-spectral quantum systems, study the supersymmetry breaking mechanism, discover new emergent hidden dynamical symmetries and find exactly or quasi-exactly solvable problems in quantum mechanics.

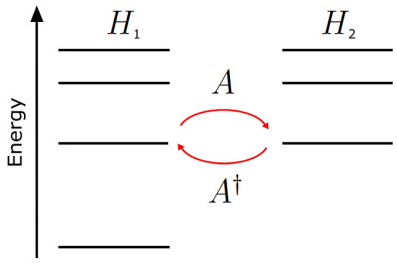

Figure 1. Energy spectra of two supersymmetric partner super-Hamiltonians and the mapping of eigenfunctions via their operators, creating additional modes. SUSY quantum mechanics contains classes of potentials in partner super-Hamiltonians with hidden symmetries, associated with nonlinear momenta integrals of motion, where coordinates and momenta variables are mixed in the phase space, generating nonlinear W-type algebras.

Various applications of supersymmetric quantum mechanics can be found in the dynamics of D-branes and black holes, M-theory and matrix models, or the theory of integrable systems and fluid mechanics. An interesting property of supersymmetry is that it manifests as a non-linear symmetry of a bosonic system without fermion degrees of freedom. By employing generalized statistics, nonlinear supersymmetry [2] can be realized without fermions. However, the quantization of nonlinear supersymmetry faces the problem of quantum anomalies. Our study is motivated by the existence of nonlinear supersymmetry in parabosonic systems, the hidden symmetries associated with their super-Hamiltonians and the emergence of quantum anomalies breaking such symmetries.

Results and discussion

Quantum analogues of classical systems with nonlinear supersymmetry face the challenge of quantum anomalies, exhibiting a number of features: an exact symmetry in the classical theory, a divergence in the quantum theory and a weak violation of the original symmetry in the regularized version of the quantum theory. The quantization of systems with nonlinear supersymmetry is a nontrivial task due to the presence of quantum anomalies. We consider nonlinear supersymmetry of one-dimensional systems with hidden symmetries generated by supercharges of higher order in momentum and study the quantum anomaly problem for such systems. We investigate properties of one-dimensional systems with nonlinear supersymmetry, finding two types of anomaly-free quantum systems with nonlinear supersymmetry.

References

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