Approximate Solutions to Axial Vibrations of Nanobars in Nonlinear Elastic Medium

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Abstract. The present contribution aims to provide some approximate solutions in closed-form to axial vibrations of nanobars modeled with Eringen's two-phase local/nonlocal model. The solution is based on formal series expansion of field functions depending on fraction coefficient representing the contribution of non-local part in constitutive equation. Some preliminary results are presented and possible next steps are discussed.

Introduction

If the characteristic external length of the structure is comparable to that of internal, predictions of classical elasticity may be erroneous. In such cases, nonlocal theories of elasticity shall be used, which are able to capture spatial wave dispersion and take into account the internal organization of the material. Among strong and weak types of nonlocal models [1] we use Eringen's strong non-local theory to examine axial vibrations of nanobars which are embedded in an elastic medium, the stiffness of which is amplitude-dependent. There are similar works in the literature utilizing differential form of Eringen's model, such as [2], which requires so-called constitutive boundary conditions and may lead to non-physical conclusions [3, 4]. With a perturbation technique which is free of any spurious additional constraints [5] we will re-visit this problem with consideration of possible nonlinearity due to surrounding medium:

$$N' = M\ddot{u} + k(u + \beta u^3), \quad N = B\left[(1-\xi)u' + \xi K * u'\right]; \quad \{u, N\} = \{u(z), N(z)\}\, f(t) \tag{1}$$

where N and u are axial force and axial displacement, B is the axial stiffness, M is the translational inertia of the section, k and β are elastic parameters of the surrounding medium, ξ is the fraction coefficient of the non-local part introducing the distant interactions through a convolution, $K * u'$. Applying Galërkin technique with a harmonic function, $f(t) = \cos\omega t$, the frequency of which depends on the non-local fraction coefficient, the nonlinear governing equation is reduced into,

$$4A_3/4 + A_2 - \lambda A_1 = 0, A_1 = M/Bu,\ A_2 = ku/B - (1-\xi)u'' - \xi(K * u')',\ A_3 = \beta k u^3/B, \lambda = \omega^2 \tag{2}$$

Assuming the displacement field and eigenvalue depends regularly on the fraction coefficient, Eq. 2 can be broken into a set of differential equations, the zeroth-order of which is that of local elastic bar. Using boundary conditions along with Fredholm compatibility equation provides the higher order vibration frequencies depending on the amplitude, and the parameters tuning the non-local characteristics of the material.

Results and Discussion

Amplitude-dependent natural frequencies are recovered, contributions of long-range interactions to the natural frequency is obtained in closed-form, which may serve for identification of material properties. Increasing vibration amplitude brings the possibility of buckling, which is in course and to be published elsewhere.

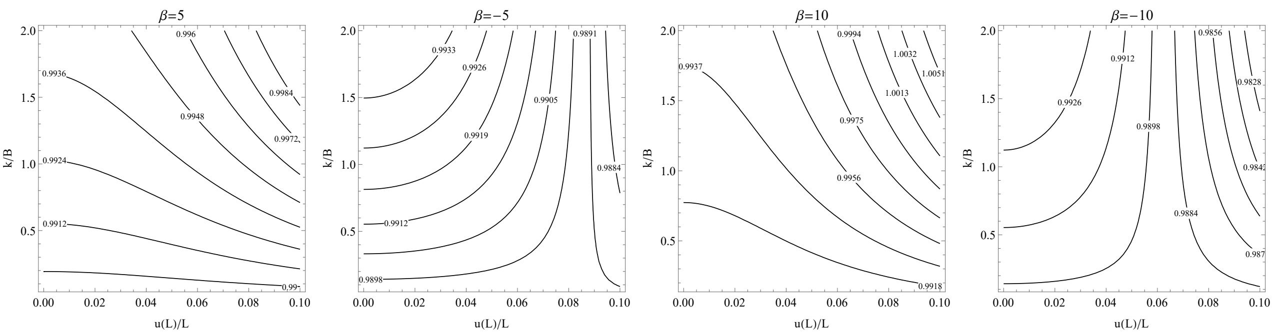

Figure 1: Iso-frequency curves, non-dimensionalized by that of local case without surrounding elastic medium. $\kappa/L = 0.1$, $\xi = 0.1$

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