Chemical Signalling and Pattern Formation in Schoener's Predator–Prey Model

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Abstract. This work presents a mathematical model to understand the impact of chemical induced defense mechanism in a special class of predator-prey model. Based on the real world observations [1, 2], we formulate a predator-prey model that considers movement of prey individuals opposite to the gradient of the chemical released by predators. We theoretically able to show existence of solutions in one dimension ($n = 1$) and global-existence of solutions for $n \geq 2$ is still a topic of debate. An onerous linear stability analysis shows that repulsive indirect predator-taxis promotes instability in proposed model and space inhomogeneous Hopf-bifurcation may occur at the critical values of taxis sensitivity parameter χ. We numerically show emergence of spatio-temporal patterning that depicts the tendency to spatio-temporal separation between prey and predators.

Introduction

Let $P(x,t)$, $N(x,t)$ represent the population densities of predator and prey individuals in a domain $\Omega \in \mathbf{R}^n$ with smooth boundary, and $C(x,t)$ is the concentration of chemical released by predators then predator-prey model studied in this paper reads

$$\begin{aligned} \frac{\partial P}{\partial t} &= d_1 \Delta P + \mathcal{F}(N,P), && x \in \Omega,\ t > 0, \\ \frac{\partial N}{\partial t} &= d_2 \Delta N + \chi \nabla.(N \nabla W) + \mathcal{G}(N,P), && x \in \Omega,\ t > 0, \\ \frac{\partial C}{\partial t} &= d_3 \Delta C + \sigma P - \mu C, && x \in \Omega,\ t > 0, \end{aligned} \tag{1}$$

with following non-zero initial condition and homogeneous Neumann boundary condition

$$\begin{aligned} P(x,0) = P_0(x),\ P(x,0) = N_0(x),\ C(x,0) = C_0(x) \qquad & x \in \Omega, \\ \partial_\nu P = \partial_\nu N = \partial_\nu C = 0 \qquad & x \in \partial\Omega,\ t > 0. \end{aligned} \tag{2}$$

In model (1), $d_i,\ i = 1, 2, 3$ are diffusion coefficients, χ counts repulsive chemo-sensitivity of prey, σ and μ represent chemical production and degradation rate, respectively. We consider Schoener's type [4] kinetic part in (1) with the assumption that both predator and prey exploit competitively a common food resource which is available at some constant rate and shared between the predator and the prey.

Discussion

The mathematical model studied in this paper reveals the defence mechanism by means of chemical signalling. Modeling framework considers a predator-prey model that counts intaguild predation and repulsive taxis opposite to gradient of chemical released by predator. We have proved the existence of global solutions in 1D and by linear stability analysis we show that sufficiently large chemosensitivity gives rise to emergence of inhomogeneous space-time patterns. Numerically we observed beautiful spatio-temporal patterns in 1D indicating that predators and prey have used the same territory at different times in what is called spatio-temporal separation in ecology.

References

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