Doubly periodic solutions and breathers of the Hirota equation: Cascading mechanism and spectral analysis

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Abstract. We take the Hirota equation which is a further modification of the nonlinear Schrödinger equation with additional terms that are responsible for third-order dispersion and a correction to the cubic nonlinearity. 'No-phase-shift' and 'phase-shifted' doubly periodic solutions for Hirota equation are obtained by theta and elliptic functions. Cubic dispersion preserves numerical robustness, since slightly disturbed exact solutions as initial states still evolve to the analytical configurations. A plane wave with a cosine wave perturbation will trigger repeated occurrence of breathers, i.e., a manifestation of the Fermi-Pasta-Ulam-Tsingou recurrence. At the formation time of the breathers, the profiles of the numerical breathers agree well with the exact doubly periodic solutions. The spectra are studied analytically and computationally, which provide the motivation for introducing 'cascading mechanism'. This mechanism elucidates the dynamics leading to the first formation of breathers.

Introduction

The nonlinear Schrödinger equation is a widely applicable model in elucidating the evolution of wave systems in many physical disciplines, e.g. fluid mechanics and optics [1, 2]. Breathers are pulsating modes, and rogue waves are surprisingly large displacements from a tranquil background. These modes are closely related to the modulation instability of plane waves. This instability arises from the interplay of dispersion and nonlinearity, and will lead to growth of small amplitude disturbances. Typically, these breathers decay after attaining maximum displacement. On reaching sufficiently small amplitude, nonlinear effect is triggered again, leading to the occurrence of breathers for the second time [3]. This cycle is repeated, and is taken as one manifestation of the Fermi-Pasta-Ulam-Tsingou recurrence (FPUT) in classical physics. Our objective is to extend this type of FPUT analysis to higher order members of the Schrödinger family of evolution equations.

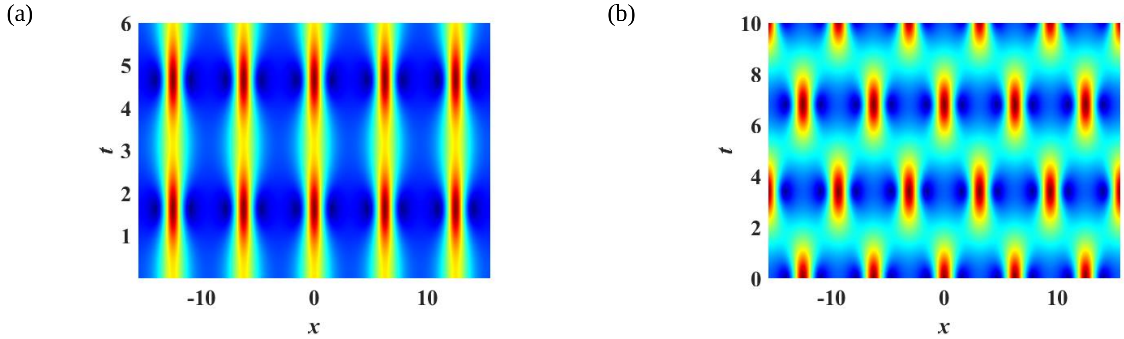

Figure 1 (a) 'No-phase-shift' and (b) 'phase-shifted' doubly periodic solutions

Results and Discussion

The properties and dynamics of breathers and doubly periodic solutions of the Hirota equation are studied. Doubly periodic solutions are established by utilizing the bilinear derivatives of theta functions. Theoretically, we expect modes similar to the Kuznetsov-Ma and Akhmediev breathers will result by long wave limit. Instead, we give a quick derivation in terms of a Darboux transformation and concentrate on the cascading mechanism and spectral analysis. The cascading mechanism can elucidate the first formation of a breather. More precisely, higher order harmonics exponentially small initially can nevertheless be amplified at a higher rate. Eventually, all these modes attain roughly the same magnitude at one instant of time (or one spatial location in the optical fiber setting), which signifies the first occurrence of a breather. We substantiate these assertions with a spectral analysis.

References

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[3] Chin, S.A., Ashour, O.A., Belić, M.R. (2015) Anatomy of the Akhmediev breather: Cascading instability, first formation time, and Fermi-Pasta-Ulam recurrence. *Phys. Rev. E* **92**: 063202.