Fuzzy Generalized Cell Mapping with Adaptive Interpolation (FGCM with AI) for Bifurcation Analysis of Nonlinear Systems with Fuzzy Uncertainties

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Abstract. Fuzzy Generalized Cell Mapping (FGCM) method is developed with the help of the Adaptive Interpolation (AI) in the space of fuzzy parameters. The adaptive interpolation on the set-valued fuzzy parameter is introduced in computing the one-step transition membership matrix to enhance the efficiency of the FGCM. For each of initial points in the state space, a coarse database is constructed at first, and then interpolation nodes are inserted into the database iteratively each time errors are examined with the explicit formula of interpolation error until the maximal errors are just under the error bound. With such an adaptively expanded database on hand, interpolating calculations assure the required accuracy with maximum efficiency gains. The new method is termed as Fuzzy Generalized Cell Mapping with Adaptive Interpolation (FGCM-AI), bifurcation analysis shows that the FGCM with AI has a thirtyfold to fiftyfold efficiency over the traditional FGCM to achieve the same analyzing accuracy.

Introduction

Fuzzy Generalized Cell Mapping with Adaptive interpolation (FGCM with AI) is introduced first. The fuzzy master equation is given as follows for the possibility transition of continuous fuzzy process,

$$p(\mathbf{x},t) = \sup_{\mathbf{x}_0 \in \mathbf{D}}[\min\{p(\mathbf{x},t|\mathbf{x}_0,t_0),\ p(\mathbf{x}_0,t_0)\}], \quad \mathbf{x} \in \mathbf{D} \tag{1}$$

where $\mathbf{x}$ is a fuzzy process, $p(\mathbf{x}, t)$ is the membership distribution function of $\mathbf{x}$ at t, and $p(x, t, x_0, t_0)$ is the transition possibility function. Equation (1) of the FGCM can be viewed as a discrete representation of fuzzy master equations. A partial differential equation for the fuzzy master equation of continuous time processes, which is analogous to the Fokker-Planck-Kolmogorov equation for the probability density function of stochastic processes. The solution to the equations is in general very difficult to obtain analytically. The FGCM offers a very effective method for solutions to fuzzy master equations, particularly, for fuzzy nonlinear dynamical systems. For the deterministic system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, the Interpolated Cell Mapping method is proposed to improve the efficiency of direct numerical simulation. It estimates the solution $\mathbf{x}(T, \mathbf{x}_0)$ by interpolating in the state space. For fuzzy system, since the solution of the equation is continuous with respect to the fuzzy parameter $s \in$ supp (S), interpolation in the fuzzy parameter space is considered to estimate the solution $\mathbf{x}(T, \mathbf{x}_0, s)$, which is expected to increase the efficiency of the FGCM[1].

Results and discussion

A 3-D jerk system with fuzzy uncertainty is considered here with the help of the FGCM with AI

$$\dddot{x} \in -\ddot{x} - \sigma\gamma\dot{x} + \sigma x - \sigma[S]^{\alpha}x^2 - \sigma x^3$$

where S is a triangular fuzzy number. In the following analyses using the FGCM with AI, the domain D = $[-1.5, 1.5] \times [-3, 3] \times [-0.8, 0.8]$ is discretized into $51\times101\times27$ cells and $5\times5\times5$ points are sampled from each cell. Supp(S) is discretized into 257 equal segments. The duration of one mapping step is $T = 2\pi$[2].

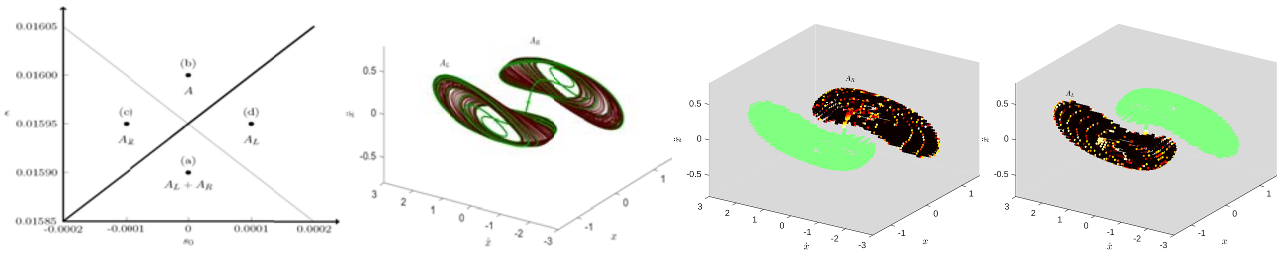

Figure 1: Codimension two bifurcation in the two parameter space of a 3-D Jerk system with fuzzy uncertainty[2]

A codimension two bifurcation of fuzzy chaotic attractors is found at a vertex in a two-parameter plane where catastrophic and explosive bifurcation curves intersect. The dynamics of the fuzzy chaotic systems is extremely rich at the vertex of the codimension two bifurcation. Such a codimension-two bifurcation is fuzzy noise-induced effects which cannot be seen in the deterministic systems.

References

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[2] Liu X.M., Jiang J., Hong L. et al. (2019) Fuzzy noise induced codimension two bifurcations captured by Fuzzy Generalized Cell Mapping with Adaptive Interpolation. *Int. J. of Bifurcation and Chaos* **29**(11): 1950151.