Nonlinear effects of the central body oblateness on the coplanar dynamics of solar sails

Martin Lara***, **Elena Fantino****, and Roberto Flores**,*

****SCoTIC – University of La Rioja, Logroño, Spain, ORCID 0000-0001-6754-916X*

***Aerospace Engineering Department, Khalifa University, Abu Dhabi, UAE, ORCID 0000-0001-7633-8567*

**Centre Internacional de Mètodes Numèrics en Enginyeria (CIMNE), Barcelona, Spain, ORCID 0000-0001-6027-9515*

Abstract. The nonlinearities introduced by the coupling of solar radiation pressure and oblateness perturbations are known to yield radical changes in the long-term dynamics of solar sails. After a reduction process in which short-period terms are removed by perturbation theory, we arrive to a time-dependent two degrees of freedom Hamiltonian depending on one physical and one dynamical parameter. While the reduced model is non-integrable in general, coplanar orbits constitute an integrable invariant manifold. We discuss the qualitative features of the coplanar dynamics, and find three regions of the parameters space characterized by the existence of different regimes of the reduced flow.

Introduction

The dynamics of natural and artificial bodies in the solar system is dominated by the Keplerian attraction of either the sun or a different natural solar system body. However, nonlinear perturbations introduced by different effects, like the non-centralities of the gravitational potential of the main attracting body or solar radiation pressure (SRP), may accumulate with time thus introducing notable changes with respect to the Keplerian dynamics. For objects with high area-to-mass ratio, SRP is an important effect that is fundamental in the description, for instance, of the dust dynamics in planetary rings. But it can be used too as an endless propellant for artificial satellites powered by solar sails. The effects of SRP alone are well known and had been thoroughly discussed [1]. However, the description of the coupled effect of the central body oblateness and SRP perturbations, while repeatedly reported in the literature [2, 3], to our knowledge is still incomplete.

Results and discussion

We make an additional effort in describing the nonlinear dynamics under coupled SRP and oblateness perturbations. More precisely, we focus on the coplanar manifold, in which the orbits of the sun and the massless body lie on the equatorial plane of the central body. Results are summarized in Fig. 1, where plot (b) shows three regions in the parameters plane separated by two curves that are computed analytically. The reduced flow is represented by eccentricity vector diagrams (plots (a) and (c) of Fig. 1), which are constructed as contour plots of the long-term Hamiltonian. In the region above the red line we only find a single fixed point corresponding to an elliptic orbit with the periapsis frozen at 180°, on average, where two different kinds of flow are possible (plot (a)). Namely, elliptic orbits with *oscillating* periapsis exist between the fixed point and the dashed contour through the origin, otherwise the periapsis of the orbits *rotates* traveling 360°. A saddle-node bifurcation happens when crossing the red curve in the parameters plane, and below it we find three fixed points. Two additional regions of orbits with oscillating and rotating periapsis are now possible, which are determined by the contour corresponding to the energy of the fixed point of the hyperbolic type. However, in the region between the red bifurcation line and the dashed curve in plot (b), the contour of the hyperbolic fixed point embraces the one of the circular orbits, making only three different kinds of flow possible.

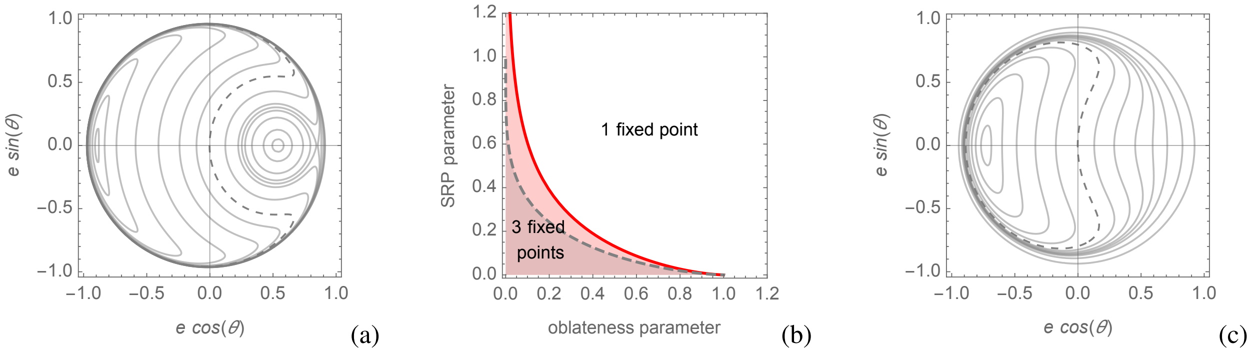

Figure 1: Qualitative dynamics of the coplanar orbits in the parameters SRP-oblateness space. θ is the periapsis-sun direction angle

In addition, we obtained the infinitesimal contact transformation that removes the short-period terms from the original Hamiltonian, in this way providing the needed mathematical support to the usual averaging assumptions used in the computation of the long-term dynamics.

References

[1] Deprit A. (1984) Dynamics of orbiting dust under radiation pressure. In: Berger A. (ed) *The big bang and Georges Lemaître*. Springer, Dordrecht, 151-180.

[2] Krivov A.V., Getino J. (1977) Orbital evolution of high-altitude balloon satellites. *A&A* **318**:308–314

[3] Alessi E.M., Colombo C., Rossi, A. (2019) Phase space description of the dynamics due to the coupled effect of the planetary oblateness and the solar radiation pressure perturbations *Celest Mech Dyn Astron*, **131**:43