A Stochastic computational technique for the multi-Pantograph-delay systems through Trigonometric approximation

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Abstract. A stochastic mathematically designed technique with logarithmic and trigonometric transformation function have been implemented for numerical treatment of multi-Pantograph differential systems. Several local optimization solvers including Interior Point, Active set, Genetic Algorithms and Sequential Quadratic Programming have been used to calculate multi-Pantograph model. Numerical experiments showed that obtained solutions through proposed scheme are better in accuracy than results presented in the literature for well-known analytical techniques, including Variational Iterative Method, Differential Transform Method and Homotopy Methods. Moreover, on the basis of several independent runs comprehensive statistical measures have been presented to endorse the validity, accuracy and reliability of the proposed scheme.

Introduction

Pantograph delay differential equations (PDDE) are characterized as functional differential equations with delays. Ockenden and Taylor [1] originated pantograph equation on collection of electric current through pantograph head of an electric locomotive. These equation are characterized by the existence of a linear functional argument. Pantograph equations have important role in explaining many differential phenomena, such as economy, probability theory, astrophysics, electrodynamics, non linear dynamic system, control theory, number theory, quantum mechanics, biological sciences and many industrial applications. Various methods have been implemented or used for solution of such equations worldwide, including recently developed methods such as Variational Iterative Method (VIM), Adomian Decomposition Method (ADM), and Differential Transform Method (DTM), etc. For detailed review the interested reader can see the articles by Yüzbaşıa and Sezer [3], Muroya et al. [4] and Li and Lua [5]. The most general form of pantograph equation [2] having with variable coefficients is given as

$$y^{(m)}(x)+\sum_{j=0}^{n}\sum_{k=0}^{m-1} y^{(k)}(\beta_{jk+\alpha_{jk}x})p_{jk}(x)+g(y)=f(x);\quad 0\le x\le b<\infty$$

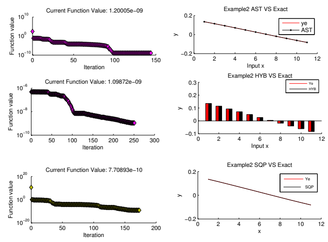

Figure 1: The results obtained through SQP, AST and HYB comparison with reference solution

Results and discussion

The developed stochastic techniques using new activation function in optimization process is executed through local and global solvers for three different problems, which provide acceptable solutions for the Pantograph-delay problems as shown in Fig.1. The proposed stochastic numerical scheme applied on pantograph delay equations to find approximate solutions matching with reference solutions with a short time management. The proposed numerical optimization solvers can easily handle the pantograph-delay equations without any restrictions and constraints on the parameters, and are promising methods for delay equations. The proposed mathematical models are having simplicity of concept, comfort of implementation, and broader applicability.

References

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