# Modelling and Analysis Measles Epidemic Model with Constant Proportional(CP) Caputo Operator

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**Abstract**. In this work, we proposed a fractional order measles model with Constant Proportional(CP) Caputo operator. We treated the proposed model's positivity, boundedness, well-posedness, and biological viability. For local and global stability analysis, we introduce the Lyapunov function with the first and second derivatives. There is a detailed discussion of additional analysis on CPC and Hilfer Generalized Proportional operators. Finally, numerical results and simulations are derived with the proposed scheme for measles model.

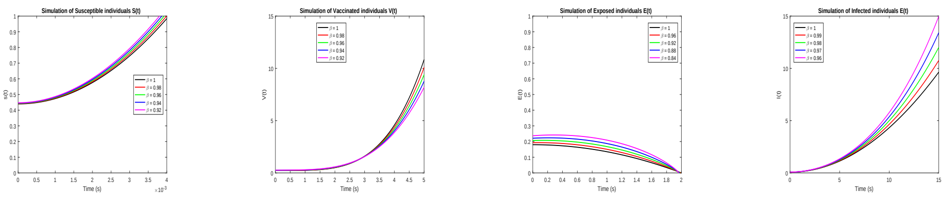
## Introduction and Fractional Order Model

Despite the fact that there is a reliable and efficient vaccine, measles, also known as rubeola, is still a major reason for death globally, particularly in developing nations. Measles is a contagious viral infection brought on by the "Paramyxovirus," a member of the Morbillivirus genus in the Paramyxoviridae family. Children under the age of five are disproportionately affected, and in 2017 the measles claimed the lives of around $110,000$ individuals, primarily young children under the age of six. Here we present a time fractional-order scheme of measles transmission dynamics. The measles transmission dynamics are mathematically modeled in [1] using a deterministic approach. The model splits the number of people into six groups, including susceptible $\mathscr{S}(t)$, vaccinated $\mathscr{V}(t)$, exposed $\mathscr{E}(t)$, infected $\mathscr{I}(t)$, hospitalized $\mathscr{H}(t)$, and recovered $\mathscr{R}(t)$, based on each person's epidemiological state. There is a rate $\phi$ of recruitment of the susceptible class on a daily basis. The aforementioned description may be expressed as a set of time-fractional order differential equations as:

$$\begin{cases} {}_0^{CPC}D_t^{\gamma}\mathscr{S}(t) = \phi - \alpha\mathscr{S}\mathscr{I} + \omega\mathscr{V} - q_1\mathscr{S}, {}_0^{CPC}D_t^{\gamma}\mathscr{V}(t) = \pi\mathscr{S} - q_2\mathscr{V}, \\ {}_0^{CPC}D_t^{\gamma}\mathscr{E}(t) = \alpha\mathscr{S}\mathscr{I} - q_3\mathscr{E}, {}_0^{CPC}D_t^{\gamma}\mathscr{I}(t) = \beta\mathscr{E} - q_4\mathscr{I}, {}_0^{CPC}D_t^{\gamma}\mathscr{H}(t) = \rho\mathscr{I} - q_5\mathscr{H}, {}_0^{CPC}D_t^{\gamma}\mathscr{R}(t) = \sigma\mathscr{H} - \mu\mathscr{R} \end{cases}$$

## Results and Conclusion

In this section, the simulation of considered model is illustrated in the figures using the value of the basic parameters from [10. Figures 1-6 show the plots for the variations of $\mathscr{S}$,$\mathscr{V}$, $\mathscr{E}$, $\mathscr{I}$, $\mathscr{H}$ and $\mathscr{R}$ for different fractional order $\gamma$. We plot the series solutions corresponding to different fractional order [2] in Figs. 1 using Mathematica. We observe that the fractional order SVEIHR measles model has more degree of freedom as compared to ordinary derivatives.



(a) $S(t)$ (b) $V(t)$ (c) $E(t)$ (d) $I(t)$

Figure 1: Dynamics of the Measles Model with proposed fractional operator

The proposed sachems qualitative and quantitative analysis are also covered. Using the Lyapunov function, we also explore local and global stability analysis. We employed various methods to invert the PC and CPC operators in order to assess the fractional integral operator. We also derived the eigenfunctions of the CPC operator from the fractional differential equations of our proposed model. Additional analysis on the CPC and Hilfer Generalized Proportional operators is covered in great detail. Create a numerical simulation of a system of fractional differential equations employing the LADM. To simulate the outcomes for different fractional order and fractal dimension values, we used Matlab.

## References

[1] James Peter, O., Ojo, M. M., Viriyapong, R., Abiodun Oguntolu, F. (2022). Mathematical model of measles transmission dynamics using real data from Nigeria. Journal of Difference Equations and Applications, 1-18.

[2] Baleanu, D., Fernandez, A., Akgl, A. (2020). On a fractional operator combining proportional and classical differintegrals. Mathematics, 8(3), 360.