Finite-size effects in neural mass models

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Abstract. Neural mass models is a general name for various models describing the collective dynamics of large neural populations in terms of averaged macroscopic variables. Recently, the so-called next-generation neural mass models attracted a lot of attention due to their ability to account for the degree of synchrony. Being exact in the limit of infinitely large number of neurons, these models provide only approximate description of finite-size networks. In the present Letter we study finite size effects in collective behaviour of neural networks and prove that these effects can be captured by appropriately modified neural mass models. Namely, we show that the finite size of the network leads to the emergence of the so-called shot noise appearing as a stochastic term in the neural mass model. The power spectrum of this shot noise contains pronounced peaks, therefore its impact on the collective dynamics might be crucial due to resonance effects.

Introduction

Electrical activity of neuronal populations provides a substrate for information processing and cognitive functions in central neural system. Mathematical modelling has been a guide on this way for more than 30 years. One of the promising approaches in mathematical modeling of neural networks is the development of reduced models describing large populations of coupled neurons in terms of low-dimensional dynamical systems for the averaged variables. Such macroscopic or "neural mass" models can be obtained heuristically or derived from the microscopic dynamics using the refractory density approach, master equation formalism or other techniques. Recently, the so-called next generation of neural mass models won much attention of the researchers. The theoretical ground for this type of models is provided by the application of Ott-Antonsen theory to populations of theta-neurons or quadratic integrate-and-fire neurons [1]. A distinctive feature of these models is their capability to account for the degree of synchrony in neuronal populations. Next-generation models were proved useful in a number of contexts including the , whole-brain simulations, etc.

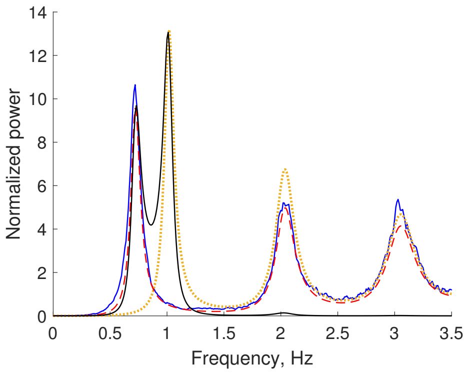

Figure 1: Power spectrum of the shot noise of a network from Ref. [1] with the parameters $J = 10$, $\bar{\eta} = 0$, $\Delta = 1$. Red dashed line: theoretical predictions. Blue solid line: numerical results for $N = 10^4$. Ocher dotted line: the noise of the uncoupled network $\chi_0(t)$. Black solid line: coupling-induced fluctuations.

Results and discussion

Being exact in the thermodynamic limit, neural mass models are considered as a good proxy of finite neuronal populations of sufficiently large size. However, whether and to what extend the population dynamics is amendable to finite-size effects is an open question. We address this point and consider the effect of the network size on the output signal it generates [2]. We demonstrate that in analogy to electronic circuits, the discrete rather than continuous nature of neurons constituting the network leads to the emergence of the shot noise. We show that adding this noise to the neural mass model transforms the latter into a system of stochastic differential equations reproducing the dynamics of a finite-size population. The core ingredient of the theory is calculation of the power spectrum of the shot noise, which consists of the shot noise for an uncoupled network plus macroscopic fluctuations induced by coupling, as demonstrated in Fig. 1. Our results allow to obtain a modified neural-mass model in the form of a stochastic differential equations which describes the coarse-grained dynamics of the finite-size network.

References

[1] Montbrió E., Pazó D., Roxin A. (2015) Macroscopic description for networks of spiking neurons. *Physical Review X*, **5**:021028.
[2] Klinshov V., Kirillov S. (2022) Shot noise in next-generation neural mass models *arXiv preprint*, arXiv:2205.01984.