An improved variable-coefficient harmonic balance method for quasi-periodic solutions

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Abstract. The variable-coefficient harmonic balance method (VCHBM) approximates quasi-periodic solutions by using a variable-coefficient Fourier series. In this work, VCHBM for tracking the quasi-periodic solutions with two irrational frequencies is improved from two aspects. First, a new formulation for alternating Frequency-Time method is proposed for VCHBM so that the time-consuming symbolic operations are avoided and the computation efficiency is enhanced. Secondly, a phase condition and a frequency condition are introduced into the arc-length continuation method to make VCHBM robust and effective. Numerical examples show the validity of the proposed improvements.

Introduction

Quasi-periodic responses widely exist in nonlinear dynamical systems, which arise under a quasi-periodic excitation or even under a single frequency excitation. Harmonic balance method (HBM) has been successfully applied to evaluate the periodic motions of nonlinear dynamical systems. Recently, multi-harmonic balance method (MHBM) [1] and variable-coefficient harmonic balance method (VCHBM) [2] are proposed as the extensions of HBM to quasi-periodic solutions. Since all the frequencies base are the prerequisite for MHBM, which is often unrealistic under a single frequency excitation. In comparison, VCHBM is relative robust in evaluating such a quasi-periodic response. However, the ways in implementing Alternating time-frequency method (AFT) and arc-length continuation (ALC) in VCHBM have flaws. Thus, improvements are proposed in order to make VCHBM more efficient and robust.

Results and discussion

For a non-autonomous nonlinear system with n-DOFs governed by the differential equations:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)+\mathbf{f}_{nl}(\mathbf{x},\dot{\mathbf{x}})-\mathbf{e}(\boldsymbol{\omega},t)=\mathbf{0}\ .$$

The quasi-periodic responses with two irrational frequencies are approximated by variable-coefficients harmonic balance method with a Fourier series $\mathbf{x}(\omega_1 t,\omega_2 t)=\mathbf{X}_0^0(\omega_2 t)+\Sigma_{j=1}^{\mathrm{H}_{\mathrm{VC}}^1}\mathbf{X}_{k_j^1}^c(\omega_2 t)\cos(k_j^1\omega_1 t)+\mathbf{X}_{k_j^1}^s(\omega_2 t)\sin(k_j^1\omega_1 t)$, where $\mathbf{X}(\omega_2 t)=\mathbf{Z}_0^0+\Sigma_{i=1}^{\mathrm{H}_{\mathrm{VC}}^2}\mathbf{Z}_{k_i^2}^c\cos(k_i^2\omega_2 t)+\mathbf{Z}_{k_i^2}^s\sin(k_i^2\omega_2 t)$. By applying the harmonic balancing in term ω_1 and ω_2 sequentially, the differential equations are eventually transformed into the $n\mathrm{L}_{\mathrm{VC}}^1\mathrm{L}_{\mathrm{VC}}^2$-dimensional nonlinear algebraic equations in the frequency domain as $\mathbf{R}(\mathbf{Z},\omega_1,\omega_2)=\mathbf{A}_{\mathrm{VC}}(\omega_1,\omega_2)\mathbf{Z}+\mathbf{F}(\mathbf{Z})-\mathbf{E}_{\mathrm{VC}}=\mathbf{0}$ [2].

In order to enhance the efficiency of AFT in determining the Fourier coefficients of the nonlinear forces, a procedure of two-step DFT and two-step inverse-DFT are proposed as shown by Fig.1(a). Two transformation matrices $\boldsymbol{\Gamma}_{\mathrm{VC}}^2$ and $\boldsymbol{\Gamma}_{\mathrm{VC}}^1$ of DFT and their inverse of *i*-DFT are updated at each step of the iterations in order to avoid the time-consuming symbolic operations. In order to achieve a robust solution track of quasi-periodic responses with respect to a free parameter, after two steps of tangent prediction and orthogonal correction of ALC are briefly reviewed, two supplemented constraint conditions, namely frequency condition and phase condition, are introduced (see Fig.1b). Especially, the phase condition $\left[\left((\nabla_2\otimes\mathbf{I}_{n\mathrm{L}_1})\mathbf{Z}\right)^T+\left((\nabla_2\otimes\mathbf{I}_{n\mathrm{L}_1})^2\mathbf{Z}\right)^T(\nabla_2\otimes\mathbf{I}_{n\mathrm{L}_1})\right]\Delta\mathbf{Z}=0$ based on an alternate phase condition [3] guarantees a unique quasi-periodic solution with a fixed phase shift and makes VCHBM robust and effective.

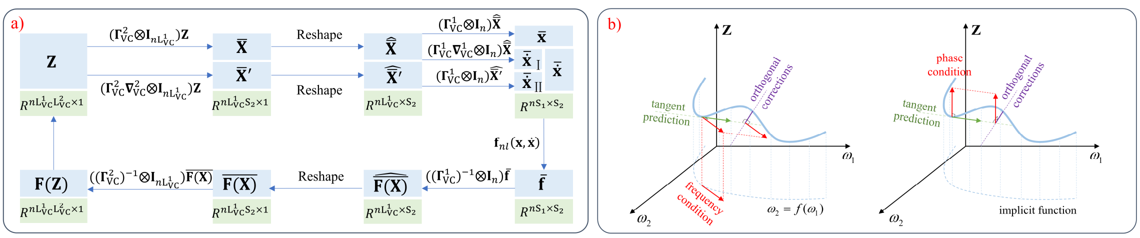

Figure 1: a) A segment after segment processing technique for AFT; b) A phase condition and a frequency condition for ALC

A SDOF Duffing oscillator system with a quasi-periodic excitation and a 2DOFs nonlinear energy sink (NES) system with a single frequency excitation are studied as examples to show the feasibility of the proposed new formulation for AFT and a more reasonable supplemented constraint condition for solution continuation.

References

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