Rolling a heavy ball on a revolving surface

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Abstract. 5 theorems on the properties of newly constructed orthogonal curvilinear coordinate systems on various revolving surfaces and 10 theorems on the properties of the non-linear dynamics of non-slip rolling of a heavy, homogeneous and isotropic ball on revolving surfaces are defined. For two special cases, when the revolving surfaces were created by the rotation of a parabola, that is, a biquadratic parabola, the corresponding nonlinear differential equations of rolling, without sliding, a heavy homogeneous and isotropic ball, as well as equations of phase trajectories, were derived. It is shown that for such nonlinear rolling dynamics there is a cyclic integral, as well as one cyclic coordinate in all cases of revolving surfaces.

Introduction

The rolling of a ball on curvilinear paths and surfaces has attracted the attention of many researchers since ancient times, both in a scientific-theoretical approach, but also in the interest of application in engineering. There are contemporary authors who claim in their works that the rolling of a ball on a surface is a system with non-holonomic connections, and the author of this paper has shown in her works [1] that the constrains are holonomic and purely geometric and that rolling, without slipping, a heavy homogeneous and isotropic ball on the surface, is a system with two degrees of freedom of rolling and with pure geometric connections. Now let's cite one doctoral dissertation, which deals with a special and interesting complex mechanical system of rolling a ball, which contains a gyroscope [2]. The dissertation was done under the mentorship of Anton Bilimović, and defended, back in 1924, before a commission that included prominent scientists Milutin Milanković, the author of the famous work "The Canon of Sun Insolation", and Mihailo Petrivić, the founder of the Serbian School of Mathematics and one of the three Doctoral students of the famous Julius Henri Poincaré.

In this work we present newly constructed orthogonal curvilinear coordinate systems over various revolving surfaces (see Fig.1.b*), as well as the nonlinear dynamics of rolling, without sliding, a heavy, homogeneous and isotropic ball on revolving surfaces (see Fig.1.c* and d*).

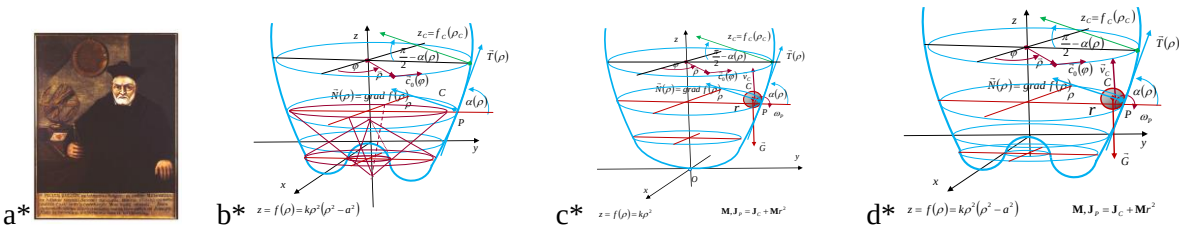

Figure 1. a* Paul Guldinu (1577 - 1643); b* Coordinate surfaces and coordinate lines of a curvilinear orthogonal coordinate system constructed over a revolving biquadratic parabolic surface; kinetic parameters of rolling a heavy ball on a revolving parabolic (c*) and biquadratic (d*) parabolic surface.

Main results and discussion

A series of nonlinear differential equations with first integrals and 15 theorems of a rolling ball, without slipping along revolving surface, are derived, and here we state the important theorem 7. The velocity $\bar{v}_C$ of the center of mass of a rolling ball, without slipping, has two components $\bar{v}_{C,m}$ and $\bar{v}_{C,c}$: one $\bar{v}_{C,m}$ in the meridional plane, parallel to the tangent $\vec{T} = \frac{1}{\sqrt{1+4k^2\rho^2}}\left(\vec{\rho}_0 + 2k\rho\vec{k}\right)$ to the derivative of the revolving parabolic (or general revolving) surface on which the ball rolls, without slipping, and the other component $\bar{v}_{C,c}$ in the circular direction $\vec{c}_0 = -\sin\varphi\vec{i} + \cos\varphi\vec{j}$, which is in the direction tangent to the circle - the curvilinear coordinate line of the parabolic revolving surface.

References

[1] Hedrih (Stevanović) R. K. (2019) Rolling heavy ball over the sphere in real Rn3 space. *Nonlinear Dyn* **97**:63-82.

[2] Demčenko V. (1923) Kotrljanje bez klizanja giroskopske lopte po sferi (Rolling without slipping a gyroscopic ball on a sphere). PhD dissertation, University of Belgrade.