Fast M-Component Direct and Inverse Nonlinear Fourier Transform

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Abstract. In this work, I demonstrate how to develop low-complexity algorithms for direct/inverse M-component nonlinear Fourier Transform (NFT) using exponential integrators.

Introduction

A nonlinear generalization of the conventional Fourier transform, referred to as *nonlinear Fourier transform* (NFT), can be achieved via the M-component Zakharov-Shabat (ZS) scattering problem which can be stated as follows [1, 2]: Let $M = \nu + 1$. For any complex-valued signal $\mathbf{q} \in \mathbb{C}^{\nu\times 1}$ ($\mathbf{r} = -\mathbf{q}^*$) and for $\zeta \in \mathbb{R}$, let $u \in \mathbb{C}^{(\nu+1)\times J}$ with

$$u_t = -i\zeta\sigma u + U(t)u \quad \text{where } \sigma = \begin{pmatrix} 1 & \mathbf{0}_\nu^\intercal \\ \mathbf{0}_\nu & -I_{\nu\times\nu} \end{pmatrix} \text{ and } U = \begin{pmatrix} 0 & \mathbf{q}^\intercal \\ \mathbf{r} & 0_{\nu\times\nu} \end{pmatrix}, \tag{1}$$

$\mathbf{0}_\nu \in \mathbb{R}^{\nu\times 1}$ and $0_{\nu\times\nu} \in \mathbb{R}^{\nu\times\nu}$ have all entries zero and $I_{\nu\times\nu} \in \mathbb{R}^{\nu\times\nu}$ is the identity matrix. Here 'ζ' is referred to as the *spectral parameter* and $U(t)$ is interpreted as the *scattering potential*. The solution of the ZS scattering problem consists in finding the so-called *scattering coeffcients* as a function of the spectral parameter ζ. A schematic of the transform is presented in Fig 1.

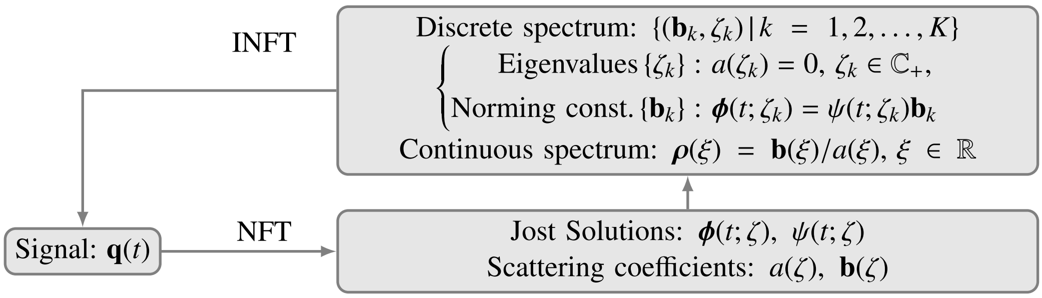

Figure 1: The figure shows a schematic of the nonlinear Fourier transform (NFT) and the inverse NFT (INFT).

The ZS scattering problem is an extremely powerful tool in studying nonlinear phenomena in nature, specially the class of initial value problems that are integrable via inverse scattering transforms. Our motivation to consider this problem stems from the recent surge of interest in NFTs for optical fiber communication (OFC) at higher signal powers. It turns out that encoding/decoding information using direct/inverse NFT can potentially mitigate the nonlinear cross-talk at higher signal powers in OFC which seem to limit the capacity of the current system. Let us note that this is the most general version of NFT which allows for larger degrees of freedom to be exploited: (a) polarization degrees of freedom (which corresponds to $M = 3$) (b) mutiple fiber modes (which corresponds to $M > 2$) [3].

The Algorithm

The case $M = 2$ [1] was treated by the author in two papers [5, 4] where algorithms for NFT/INFT with complexity $O(NK + N\log^2 N)$ and order of convergence $O(N^{-2})$ was presented (K: number of eigenvalues, N: number of samples of the signal). For the direct NFT, the determination of eigenvalues is not included in the complexity estimate. In this work, we extend these results to $M > 2$ using exponential integrators of order 2. The discrete system thus obtained happens to have the so-called *layer-peeling* property which plays a key role in the development of the new algorithms. Further, the transfer matrices have the familiar polynomial structure which makes it amenable to FFT-based fast polynomial multiplication. The mathematical details will be presented in a full-length paper.

References

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