The predictable chaos of rare events in complex systems

Tommaso Alberti[*], Davide Faranda[**], Valerio Lucarini[***]
[*]*Istituto Nazionale di Geofisica e Vulcanologia, Rome, Italy, ORCID 0000-0001-6096-0220*
[**]*Laboratoire des Sciences du Climat et de l'Environnement, Gif-Sur-Yvette, France, ORCID 0000-0001-5001-5698*
[***]*University of Reading, Reading, United Kingdom, ORCID 0000-0001-9392-1471*

Abstract. Many natural systems show emergent phenomena at different scales, leading to scaling regimes with signatures of chaos at large scales and an apparently random behavior at small scales. These features are usually investigated quantitatively by studying the properties of the underlying attractor. This multi-scale nature of natural systems makes it practically impossible to get a clear picture of the attracting set as it spans over a wide range of spatial scales and may even change in time due to non-stationary forcing. Here we present a review of some recent advancements in characterizing the number of degrees of freedom and the predictability horizon of complex systems showing non-hyperbolic chaos, randomness, state-dependent persistence and predictability. We compare classical approaches, based on Lyapunov exponents and correlation dimension, with novel frameworks based on combining adaptive decomposition methods with concepts from extreme value theory.

Introduction

Complex systems are made of different nonlinearly interacting intrinsic and extrinsic components resulting in various positive and negative feedbacks that can lead to the emergence of unpredictable temporal dynamics, or the ability of systems to spontaneously form temporal, spatial, or spatiotemporal patterns. Since 1960s complex systems have been studied in the framework of dissipative dynamical systems with the development of measures to quantify the topology of the state-space trajectories [1] and in revising some earlier concepts on their forecast horizon [2]. A one-parametric family of measures, the so-called generalized fractal dimensions, has been proposed based on a coarse-grained invariant measure linking the geometric properties of the state-space trajectories to the statistics of the dynamical scaling properties [3]. However, for systems exhibiting heterogeneous state-space structure or even non-stationarity, it would be useful to track the instantaneous number of degrees of freedoms, which are closely related to the predictability of the system and its associated recurrence characteristics [4]. The purpose of this study is to thoroughly extend an existing formalism of multi-scale measures [5] to characterize the instantaneous scale-dependent properties of complex systems by combining time series decomposition methods with concepts from extreme value theory that are related to the instantaneous number of degrees of freedom of the observed dynamics [6].

Results and discussion

Our results show that the newly introduced formalism, based on instantaneous scale-dependent dimensions, allows us to discern two properties that are inaccessible by previous global or scale-dependent analysis, namely the existence of different scale-dependent source processes (as the presence of noise or a dominant scale) and the structural stability of fixed points. Our study indicates that when considering different scales, the concept of a single universal attractor should be revised. Indeed, we have shown that a new structure of attractors, whose properties evolve in time, space and scale, is discovered by looking for fixed points and following their evolution from small to large scale and vice versa. Thus, the geometric structure of the attractor is gradually deformed and depends on the scale at which we are investigating the respective system. The main novelty introduced in this study is a powerful method to identify the existence of processes of different origin by looking at the spatial distribution of fractal dimensions across the full phase-space trajectories at different timescales. Our formalism also demonstrated the failure of the concept of universality of turbulent attractors since their properties depend on the scale we are focusing on. Given the changing nature of such attractors in time and scales we introduced the novel concept of *chameleon attractors* [7].

References

1. Grassberger P., Procaccia I. (1983) Characterization of strange attractors. *Phys. Rev. Lett.* **50**:346-349.
2. Kolmogorov A.N. (1959) Entropy per unit time as a metric invariant of automorphism. *Dok. Russ. Acad. Sci.* **124**:754-755.
3. Hentschel H.G.E., Procaccia I. (1983) The infinite number of generalized dimensions of fractals and strange attractors. *Physica D Nonlinear Phenomena* **8**:435-444.
4. Faranda D., Lucarini V., Turchetti G., Vaienti S. (2012) Generalized Extreme Value Distribution Parameters as Dynamical Indicators of Stability. *Int. J. Bif. Chaos* **22**:1250276.
5. Alberti T., Consolini G., Ditlevsen P.D., Donner R.V., Quattrociocchi V. (2020) Multiscale measures of phase-space trajectories. *Chaos* **30**:123116.
6. Alberti T., Faranda D., Lucarini V., Donner R.V., Dubrulle B., Daviaud F. (2023) Scale dependence of fractal dimension in deterministic and stochastic Lorenz-63 systems. *Chaos* **33**:023144.
7. Alberti T., Daviaud F., Donner R.V., Dubrulle B., Faranda D., Lucarini V. (2023) Chameleon attractors in turbulent flows. *Chaos, Solitons & Fractals* **168**:113195.