Safe Basins of Escape of a Weakly Damped Particle From a Truncated Quadratic Potential Well Under Harmonic Excitation

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Abstract. The safe basins (SB) of escape of a weakly damped particle from a symmetrically truncated quadratic potential well under harmonic excitation are investigated. It is assumed that the excitation frequency is in the vicinity of the resonant frequency of the potential well. An analytic approach to determine the size and the location of the non-escaping set in the plane of the initial conditions (IC) depending on several model parameters is introduced.

Introduction

Escape form a potential well is a classic problem arising in numerous fields of engineering [1] and natural sciences [2]. Escape can be caused by different types of excitation starting with the appropriate initial conditions via harmonic excitation [2] to stochastic noise [1] and impact loading. Safe basins of escape describe the set of initial conditions for which the particle does not escape from the well under a certain parameter combination [3]. The determination and understanding of the parameters' impact on the SB is of great engineering importance contributing to the design of robust systems and applications. *Karmi et al.* succeeded to derive a conservation law for forced escape with the use of action angle variables and averaging for a quadratic-quartic potential truncated at different energy levels [4], the fully analytic treatment of the SB, however, is rarely possible due to the complexity of the problem. In the following, one of the exceptional cases is presented, escape of a weakly damped particle from a truncated quadratic potential well under harmonic excitation.

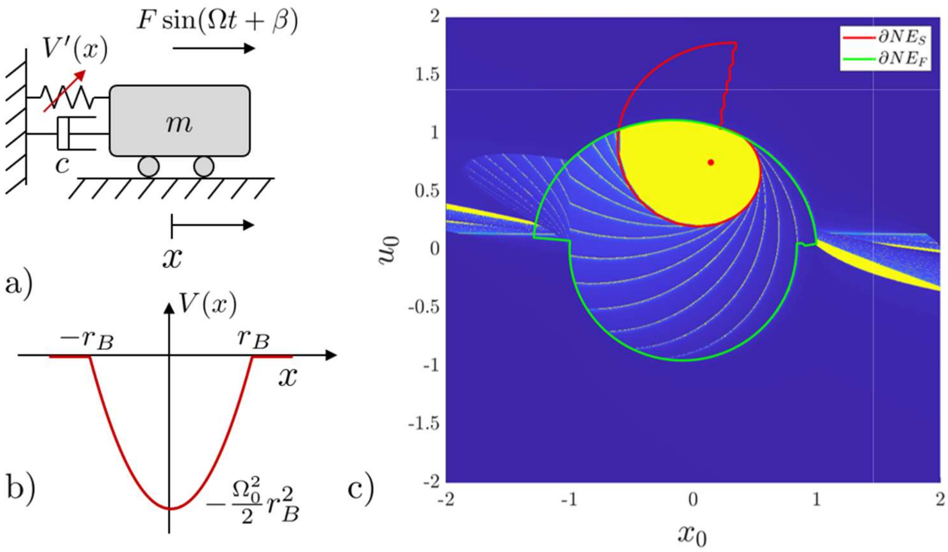

Figure 1: a) The scheme of the problem; b) the truncated quadratic potential; c) comparison of the safe basins of escape obtained numerically (yellow regions) and analytically (red and green lines referring to different escape scenarios) for some parameter setting.

Results and discussion

The solution of the equation of motion of the system shown in Fig. 1a-1b. can be obtained analytically as long as the particle is inside the well. In the analytic approach escape is identified once the particle reaches the potential boundary. It is possible to determine the envelope of the beat-like oscillating solution which leads to inequalities in polar coordinates describing the set of non-escaping coordinates in the IC plane. Two escape mechanism are identified, slow and fast escape determining together the SB (c.f. Fig. 1c). The essential properties of the SB can be described by using only two parameters. The analytic approach also explains why and how the erosion of the SB becomes more sudden if the damping coefficient is increased.

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